

This Page Is Inserted by IFW Operations  
and is not a part of the Official Record

## **BEST AVAILABLE IMAGES**

Defective images within this document are accurate representations of the original documents submitted by the applicant.

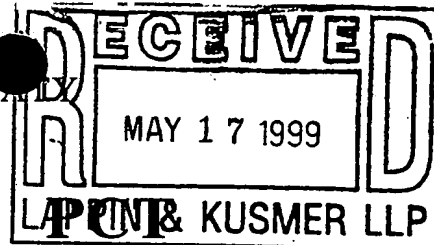
Defects in the images may include (but are not limited to):

- BLACK BORDERS
- TEXT CUT OFF AT TOP, BOTTOM OR SIDES
- FADED TEXT
- ILLEGIBLE TEXT
- SKEWED/SLANTED IMAGES
- COLORED PHOTOS
- BLACK OR VERY BLACK AND WHITE DARK PHOTOS
- GRAY SCALE DOCUMENTS

**IMAGES ARE BEST AVAILABLE COPY.**

**As rescanning documents *will not* correct images,  
please do not report the images to the  
Image Problems Mailbox.**

# PATENT COOPERATION TREATY



From the INTERNATIONAL SEARCHING AUTHORITY

To: MARK G. LAPPIN  
LAPPIN & KUSMER LLP  
200 STATE STREET  
BOSTON MA 02109

**DOCKETED**  
**PCM**

6/13/99 Comment on Abstract  
7/13/99 Resp Search Report

NOTIFICATION OF TRANSMITTAL OF  
THE INTERNATIONAL SEARCH REPORT  
OR THE DECLARATION

(PCT Rule 44.1)

Date of Mailing  
(day/month/year)

**13 MAY 1999**

Applicant's or agent's file reference

GCLL-115PC

**FOR FURTHER ACTION** See paragraphs 1 and 4 below

International application No.

PCT/US98/26384

International filing date  
(day/month/year)

**11 DECEMBER 1998**

Applicant

NESTECH INC.

1. ☒ The applicant is hereby notified that the international search report has been established and is transmitted herewith.

**Filing of amendments and statement under Article 19:**

The applicant is entitled, if he so wishes, to amend the claims of the international application (see Rule 46):

**When?** The time limit for filing such amendments is normally 2 months from the date of transmittal of the international search report; however, for more details, see the notes on the accompanying sheet.

**Where?** Directly to the International Bureau of WIPO  
34, chemin des Colombettes  
1211 Geneva 20, Switzerland  
Facsimile No.: (41-22) 740.14.35

For more detailed instructions, see the notes on the accompanying sheet.

2. ☐ The applicant is hereby notified that no international search report will be established and that the declaration under Article 17(2)(a) to that effect is transmitted herewith.

3. ☐ With regard to the protest against payment of (an) additional fee(s) under Rule 40.2, the applicant is notified that:

☐ the protest together with the decision thereon has been transmitted to the International Bureau together with the applicant's request to forward the texts of both the protest and the decision thereon to the designated Offices.

☐ no decision has been made yet on the protest; the applicant will be notified as soon as a decision is made.

4. **Further action(s):** The applicant is reminded of the following:

Shortly after 18 months from the priority date, the international application will be published by the International Bureau.

If the applicant wishes to avoid or postpone publication, a notice of withdrawal of the international application, or of the priority claim, must reach the International Bureau as provided in rules 90 bis 1 and 90 bis 3, respectively, before the completion of the technical preparations for international publication.

Within 19 months from the priority date, a demand for international preliminary examination must be filed if the applicant wishes to postpone the entry into the national phase until 30 months from the priority date (in some Offices even later).

Within 20 months from the priority date, the applicant must perform the prescribed acts for entry into the national phase before all designated Offices which have not been elected in the demand or in a later election within 19 months from the priority date or could not be elected because they are not bound by Chapter II.

Name and mailing address of the ISA/US  
Commissioner of Patents and Trademarks  
Box PCT  
Washington, D.C. 20231

Facsimile No. (703) 305-3230

Authorized officer

WILLIAM GRANT

Telephone No. (703) 308-1108

# PATENT COOPERATION TREATY

## PCT

### INTERNATIONAL SEARCH REPORT

(PCT Article 18 and Rules 43 and 44)

Applicant's or agent's file reference <b>GCLL-115PC</b>	<div style="display: flex; justify-content: space-between;"> <div style="width: 30%;">FOR FURTHER ACTION</div> <div style="width: 70%;">see Notification of Transmittal of International Search Report (Form PCT/ISA/220) as well as, where applicable, item 5 below.</div> </div>	
International application No. <b>PCT/US98/26384</b>	International filing date (day/month/year) <b>11 DECEMBER 1998</b>	(Earliest) Priority Date (day/month/year) <b>12 DECEMBER 1997</b>
Applicant <b>NESTECH INC.</b>		

This international search report has been prepared by this International Searching Authority and is transmitted to the applicant according to Article 18. A copy is being transmitted to the International Bureau.

This international search report consists of a total of 4 sheets.

☒ It is also accompanied by a copy of each prior art document cited in this report.

1. ☒ Certain claims were found unsearchable (See Box I).
  
2. ☐ Unity of invention is lacking (See Box II).
  
3. ☐ The international application contains disclosure of a nucleotide and/or amino acid sequence listing and the international search was carried out on the basis of the sequence listing
 

☐ filed with the international application.  
☐ furnished by the applicant separately from the international application,  

☐ but not accompanied by a statement to the effect that it did not include matter going beyond the disclosure in the international application as filed.

☐ transcribed by this Authority.
  
4. With regard to the title,
 

☒ the text is approved as submitted by the applicant.  
☐ the text has been established by this Authority to read as follows:
  
5. With regard to the abstract,
 

☐ the text is approved as submitted by the applicant.  
☒ the text has been established, according to Rule 38.2(b), by this Authority as it appears in Box III. The applicant may, within one month from the date of mailing of this international search report, submit comments to this Authority.
  
6. The figure of the drawings to be published with the abstract is:  
 Figure No. 1

☐ as suggested by the applicant. ☐ None of the figures.  
☒ because the applicant failed to suggest a figure.  
☐ because this figure better characterizes the invention.

# INTERNATIONAL SEARCH REPORT

International application No.  
PCT/US98/26384

## Box I Observations where certain claims were found unsearchable (Continuation of item 1 of first sheet)

This international report has not been established in respect of certain claims under Article 17(2)(a) for the following reasons:

1. ☐ Claims Nos.:  
because they relate to subject matter not required to be searched by this Authority, namely:
  
2. ☐ Claims Nos.:  
because they relate to parts of the international application that do not comply with the prescribed requirements to such an extent that no meaningful international search can be carried out, specifically:
  
3. ☒ Claims Nos.: 2 on page 46 should be deleted and replaced by "6.", and claim 6 should be "F".  
because they are dependent claims and are not drafted in accordance with the second and third sentences of Rule 6.4(a).

## Box II Observations where unity of invention is lacking (Continuation of item 2 of first sheet)

This International Searching Authority found multiple inventions in this international application, as follows:

1. ☐ As all required additional search fees were timely paid by the applicant, this international search report covers all searchable claims.
2. ☐ As all searchable claims could be searched without effort justifying an additional fee, this Authority did not invite payment of any additional fee.
3. ☐ As only some of the required additional search fees were timely paid by the applicant, this international search report covers only those claims for which fees were paid, specifically claims Nos.:
  
4. ☐ No required additional search fees were timely paid by the applicant. Consequently, this international search report is restricted to the invention first mentioned in the claims; it is covered by claims Nos.:

Remark on Protest

- ☐ The additional search fees were accompanied by the applicant's protest.  
☐ No protest accompanied the payment of additional search fees.

**Box III TEXT OF THE ABSTRACT (Continuation of item 5 of the first sheet)**

The technical features mentioned in the abstract do not include a reference sign between parentheses (PCT Rule 8.1(d)).

**NEW ABSTRACT**

A system (10) includes a part input device (12) a programmed digital computer (14) with an associated display (16) and an output layout buffer print (18). The system includes nesting process of placing objects together with minimal void space between them. The system is for defining a layout for a plurality of irregular two dimensional (2D) objects on a target plane and three dimensional (3D) objects in a target plane.

## INTERNATIO SEARCH REPORT

International application No.  
PCT/US98/26384

## A. CLASSIFICATION OF SUBJECT MATTER

IPC(6) : B26F 1/38, B28B 1/087, B65D 5/54

US CL : 364/468.04, 83/32, 264/72, 229/212

According to International Patent Classification (IPC) or to both national classification and IPC

## B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

U.S. : 364/468.04, 83/32, 264/72, 229/212

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched  
SHOE BOXElectronic data base consulted during the international search (name of data base and, where practicable, search terms used)  
APS, EPO, JPO, USCOR

## C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US 1,830,819 A (WISHMEIER) 10 November 1931, col. fig. 4	1-6
Y	US 1,942,043 A (ANDERSON) 02 January 1934, fig. 1-5	1-6
Y	US 4,838,430 A (BEEMAN et al.) 13 JUNE 1989, fig. 1-7, and col. 1-6	1-6



Further documents are listed in the continuation of Box C.



See patent family annex.

* Special categories of cited documents:	*T* later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
*A* document defining the general state of the art which is not considered to be of particular relevance	*X* document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
*B* earlier document published on or after the international filing date	*Y* document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
*L* document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)	*G* document member of the same patent family
*O* document referring to an oral disclosure, use, exhibition or other means	
*P* document published prior to the international filing date but later than the priority date claimed	

Date of the actual completion of the international search

14 APRIL 1999

Date of mailing of the international search report

13 MAY 1999

Name and mailing address of the ISA/US  
Commissioner of Patents and Trademarks  
Box PCT  
Washington, D.C. 20231

Facsimile No. (703) 305-3230

Authorized officer

WILLIAM GRANT

Telephone No. (703) 308-1108

PATENT COOPERATION TREATY REC'D 22 AUG 2000

PCT

WIPO

PCT

INTERNATIONAL PRELIMINARY EXAMINATION REPORT

(PCT Article 36 and Rule 70)

RECEIVED  
MAY 9 - 2001  
Technology Center 2169

Applicant's or agent's file reference GCLL-115PC	FOR FURTHER ACTION See Notification of Transmittal of International Preliminary Examination Report (Form PCT/IPEA/416)	
International application No. PCT/US98/26384	International filing date (day/month/year) 11 DECEMBER 1998	Priority date (day/month/year) 12 DECEMBER 1997
International Patent Classification (IPC) or national classification and IPC IPC(6): B26F 1/38, B28B 1/087, B65D 5/54 and US Cl.: 364/468.04, 83/32, 264/72, 229/212		
Applicant NESTECH INC.		

- This international preliminary examination report has been prepared by this International Preliminary Examining Authority and is transmitted to the applicant according to Article 36.
- This REPORT consists of a total of 4 sheets.  
☐ This report is also accompanied by ANNEXES, i.e., sheets of the description, claims and/or drawings which have been amended and are the basis for this report and/or sheets containing rectifications made before this Authority. (see Rule 70.16 and Section 607 of the Administrative Instructions under the PCT).  
These annexes consist of a total of 6 sheets.

- This report contains indications relating to the following items:

- I ☒ Basis of the report
- II ☐ Priority
- III ☐ Non-establishment of report with regard to novelty, inventive step or industrial applicability
- IV ☐ Lack of unity of invention
- V ☒ Reasoned statement under Article 35(2) with regard to novelty, inventive step or industrial applicability; citations and explanations supporting such statement
- VI ☐ Certain documents cited
- VII ☐ Certain defects in the international application
- VIII ☐ Certain observations on the international application

CORRECTED VERSION

Date of submission of the demand 02 JULY 1999	Date of completion of this report 19 JUNE 2000
Name and mailing address of the IPEA/US Commissioner of Patents and Trademarks Box PCT Washington, D.C. 20231	Authorized officer WILLIAM GRANT Joni Hill
Facsimile No. (703) 305-3230	Telephone No. (703) 308-1108

## INTERNATIONAL PRELIMINARY EXAMINATION REPORT

International application No.

PCT/US98/26384

**I. Basis of the report**

1. This report has been drawn on the basis of *(Substitute sheets which have been furnished to the receiving Office in response to an invitation*

- ☐ the international application as originally filed.
- ☒ the description, pages (See Attached) , as originally filed.  
pages \_\_\_\_\_ , filed with the demand.  
pages \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.  
pages \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.
- ☒ the claims, Nos. (See Attached) , as originally filed.  
Nos. \_\_\_\_\_ , as amended under Article 19.  
Nos. \_\_\_\_\_ , filed with the demand.  
Nos. \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.  
Nos. \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.
- ☒ the drawings, sheets/fig (See Attached) , as originally filed.  
sheets/fig \_\_\_\_\_ , filed with the demand.  
sheets/fig \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.  
sheets/fig \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.

2. The amendments have resulted in the cancellation of:

- ☒ the description, pages NONE.
- ☒ the claims, Nos. NONE.
- ☒ the drawings, sheets/fig NONE.

3. ☐ This report has been established as if (some of) the amendments had not been made, since they have been considered to go beyond the disclosure as filed, as indicated in the Supplemental Box Additional observations below (Rule 70.2(c)).

4. Additional observations, if necessary:

NONE



## INTERNATIONAL PRELIMINARY EXAMINATION REPORT

International application No.

PCT/US98/26384

**V. Reasoned statement under Article 35(2) with regard to novelty, inventive step or industrial applicability; citations and explanations supporting such statement****1. STATEMENT**

Novelty (N)

Claims 1-6 YESClaims NONE NO

Inventive Step (IS)

Claims 1-6 YESClaims NONE NO

Industrial Applicability (IA)

Claims 1-6 YESClaims NONE NO**2. CITATIONS AND EXPLANATIONS**

Claims 1-6 meet the criteria set out in PCT Article 33(2)-(4), because the prior art does not teach or fairly suggest steps (i), (ii) of claim 1, also (i), and (i) one (iii) of claim 3, step E, (iii), (V) of claim 5.

**Supplemental Box**

(To be used when the space in any of the preceding boxes is not sufficient)

Continuation of: Boxes I - VIII

Sheet 10

**I. BASIS OF REPORT:**

This report has been drawn on the basis of the description,  
pages, NONE, as originally filed.  
pages, NONE, filed with the demand.  
and additional amendments:  
Pages 1-39, filed with the letter of 21 July 1999.

This report has been drawn on the basis of the claims,  
numbers, NONE, as originally filed.  
numbers, NONE, as amended under Article 19.  
numbers, NONE, filed with the demand.  
and additional amendments:  
Claims 1-6, filed with the letter of 21 July 1999.

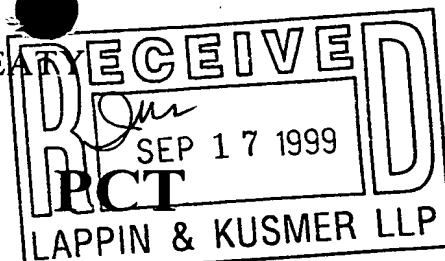
This report has been drawn on the basis of the drawings,  
sheets, NONE, as originally filed.  
sheets, NONE, filed with the demand.  
and additional amendments:  
Sheets 1-15, filed with the letter of 21 July 1999.

## PATENT COOPERATION TREATY

From the  
INTERNATIONAL PRELIMINARY EXAMINING AUTHORITY

To: MARK G. LAPPIN  
LAPPIN & KUSMER LLP  
200 STATE STREET  
BOSTON MA 02109

10/PCT Rec'd 08 JUN 2000



NOTIFICATION OF TRANSMITTAL OF  
INTERNATIONAL PRELIMINARY  
EXAMINATION REPORT

(PCT Rule 71.1)

Date of Mailing  
(day/month/year)

15 SEP 1999

Applicant's or agent's file reference

GCLL-115PC

IMPORTANT NOTIFICATION

International application No.

PCT/US98/26384

International filing date (day/month/year)

11 DECEMBER 1998

Priority Date (day/month/year)

12 DECEMBER 1997

Applicant

NESTECH INC.

1. The applicant is hereby notified that this International Preliminary Examining Authority transmits herewith the international preliminary examination report and its annexes, if any, established on the international application.
2. A copy of the report and its annexes, if any, is being transmitted to the International Bureau for communication to all the elected Offices.
3. Where required by any of the elected Offices, the International Bureau will prepare an English translation of the report (but not of any annexes) and will transmit such translation to those Offices.
4. **REMINDER**

The applicant must enter the national phase before each elected Office by performing certain acts (filing translations and paying national fees) within 30 months from the priority date (or later in some Offices)(Article 39(1))(see also the reminder sent by the International Bureau with Form PCT/IB/301).

Where a translation of the international application must be furnished to an elected Office, that translation must contain a translation of any annexes to the international preliminary examination report. It is the applicant's responsibility to prepare and furnish such translation directly to each elected Office concerned.

For further details on the applicable time limits and requirements of the elected Offices, see Volume II of the PCT Applicant's Guide.

Name and mailing address of the IPEA/US  
Commissioner of Patents and Trademarks  
Box PCT  
Washington, D.C. 20231

Facsimile No. (703) 305-3230

Authorized officer

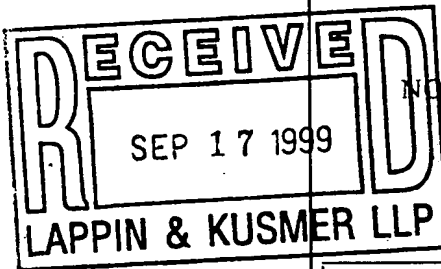
WILLIAM GRANT

Telephone No. (703) 308-1108

# PATENT COOPERATION TREATY

From the  
INTERNATIONAL PRELIMINARY EXAMINING AUTHORITY

To: MARK G. LAPPIN  
LAPPIN & KUSMER LLP  
200 STATE STREET  
BOSTON MA 02109



## PCT

NOTIFICATION OF TRANSMITTAL OF  
INTERNATIONAL PRELIMINARY  
EXAMINATION REPORT

(PCT Rule 71.1)

Applicant's or agent's file reference <b>GCLL-115PC</b>		<b>IMPORTANT NOTIFICATION</b>	
International application No. <b>PCT/US98/26384</b>	International filing date (day/month/year) <b>11 DECEMBER 1998</b>	Priority Date (day/month/year) <b>12 DECEMBER 1997</b>	
Applicant <b>NESTECH INC.</b>			

1. The applicant is hereby notified that this International Preliminary Examining Authority transmits herewith the international preliminary examination report and its annexes, if any, established on the international application.
2. A copy of the report and its annexes, if any, is being transmitted to the International Bureau for communication to all the elected Offices.
3. Where required by any of the elected Offices, the International Bureau will prepare an English translation of the report (but not of any annexes) and will transmit such translation to those Offices.
4. **REMINDER**

The applicant must enter the national phase before each elected Office by performing certain acts (filing translations and paying national fees) within 30 months from the priority date (or later in some Offices)(Article 39(1))(see also the reminder sent by the International Bureau with Form PCT/IB/301).

Where a translation of the international application must be furnished to an elected Office, that translation must contain a translation of any annexes to the international preliminary examination report. It is the applicant's responsibility to prepare and furnish such translation directly to each elected Office concerned.

For further details on the applicable time limits and requirements of the elected Offices, see Volume II of the PCT Applicant's Guide.

Name and mailing address of the IPEA/US Commissioner of Patents and Trademarks Box PCT Washington, D.C. 20231	Authorized officer <b>WILLIAM GRANT</b>
Facsimile No. (703) 305-3230	Telephone No. (703) 308-1108

# PATENT COOPERATION TREATY

## PCT

### INTERNATIONAL PRELIMINARY EXAMINATION REPORT

(PCT Article 36 and Rule 70)

Applicant's or agent's file reference GCLL-115PC	FOR FURTHER ACTION See Notification of Transmittal of International Preliminary Examination Report (Form PCT/IPEA/416)	
International application No. PCT/US98/26384	International filing date (day/month/year) 11 DECEMBER 1998	Priority date (day/month/year) 12 DECEMBER 1997
International Patent Classification (IPC) or national classification and IPC IPC(6): B26F 1/38, B28B 1/087, B65D 5/54 and US Cl.: 364/468.04, 83/32, 264/72, 229/212		
Applicant NESTECH INC.		

<p>1. This international preliminary examination report has been prepared by this International Preliminary Examining Authority and is transmitted to the applicant according to Article 36.</p> <p>2. This REPORT consists of a total of <u>4</u> sheets.</p> <p><input checked="" type="checkbox"/> This report is also accompanied by ANNEXES, i.e., sheets of the description, claims and/or drawings which have been amended and are the basis for this report and/or sheets containing rectifications made before this Authority. (see Rule 70.16 and Section 607 of the Administrative Instructions under the PCT).</p> <p>These annexes consist of a total of <u>58</u> sheets.</p>	<p>3. This report contains indications relating to the following items:</p> <ul style="list-style-type: none"> <li>I <input checked="" type="checkbox"/> Basis of the report</li> <li>II <input type="checkbox"/> Priority</li> <li>III <input type="checkbox"/> Non-establishment of report with regard to novelty, inventive step or industrial applicability</li> <li>IV <input type="checkbox"/> Lack of unity of invention</li> <li>V <input checked="" type="checkbox"/> Reasoned statement under Article 35(2) with regard to novelty, inventive step or industrial applicability; citations and explanations supporting such statement</li> <li>VI <input type="checkbox"/> Certain documents cited</li> <li>VII <input type="checkbox"/> Certain defects in the international application</li> <li>VIII <input type="checkbox"/> Certain observations on the international application</li> </ul>
---	--

Date of submission of the demand  02 JULY 1999	Date of completion of this report  22 JULY 1999
Name and mailing address of the IPEA/US Commissioner of Patents and Trademarks Box PCT Washington, D.C. 20231	Authorized officer  WILLIAM GRANT <i>For Virginia Zepa</i>
Facsimile No. (703) 305-3230	Telephone No. (703) 308-1108

## INTERNATIONAL PRELIMINARY EXAMINATION REPORT

International application No.

PCT/US98/26384

## I. Basis of the report

1. This report has been drawn on the basis of *(Substitute sheets which have been furnished to the receiving Office in response to an invitation under Article 14 are referred to in this report as "originally filed" and are not annexed to the report since they do not contain amendments):*

- ☐ the international application as originally filed.
- ☒ the description, pages (See Attached) , as originally filed.  
pages \_\_\_\_\_ , filed with the demand.  
pages \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.  
pages \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.
- ☒ the claims. Nos. (See Attached) , as originally filed.  
Nos. \_\_\_\_\_ , as amended under Article 19.  
Nos. \_\_\_\_\_ , filed with the demand.  
Nos. \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.  
Nos. \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.
- ☒ the drawings, sheets/~~fig~~ (See Attached) , as originally filed. .  
sheets/~~fig~~ \_\_\_\_\_ , filed with the demand.  
sheets/~~fig~~ \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.  
sheets/~~fig~~ \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.

2. The amendments have resulted in the cancellation of:

- ☒ the description, pages NONE .
- ☒ the claims, Nos. NONE .
- ☒ the drawings, sheets/~~fig~~ NONE .

3. ☐ This report has been established as if (some of) the amendments had not been made, since they have been considered to go beyond the disclosure as filed, as indicated in the ~~Supplemental Box~~ Additional observations below (Rule 70.2(c)).

4. Additional observations, if necessary:

NONE

## INTERNATIONAL PRELIMINARY EXAMINATION REPORT

International application No.

PCT/US98/26384

**V. Reasoned statement under Article 35(2) with regard to novelty, inventive step or industrial applicability; citations and explanations supporting such statement****1. STATEMENT**

Novelty (N)	Claims <u>NONE</u>	YES
	Claims <u>1-6</u>	NO
Inventive Step (IS)	Claims <u>NONE</u>	YES
	Claims <u>1-6</u>	NO
Industrial Applicability (IA)	Claims <u>NONE</u>	YES
	Claims <u>1-6</u>	NO

**2. CITATIONS AND EXPLANATIONS**

Claims 1-6 meet the criteria set out in PCT Article 33(2)-(4), because the prior art does not teach or fairly suggest steps (i), (ii) of claim 1, also (i), and (i) one (iii) of claim 3, step E, (iii), (V) of claim 5.

Claims 1-6 meet the criteria set out in PCT Article 33(2)-(4), because the prior art does not teach or fairly suggest steps (i), (ii) of claim 1, also (i), and (i) one (iii) of claim 3, step E, (iii), (V) of claim 5.

**Supplemental Box**

(To be used when the space in any of the preceding boxes is not sufficient)

Continuation of: Boxes I - VIII

Sheet 10

**I. BASIS OF REPORT:**

This report has been drawn on the basis of the description.  
pages, NONE, as originally filed.  
pages, NONE, filed with the demand.  
and additional amendments:  
Pages 1-39 filed with the letter of 21 July 1999.

This report has been drawn on the basis of the claims.  
numbers, NONE, as originally filed.  
numbers, NONE, as amended under Article 19.  
numbers, NONE, filed with the demand.  
and additional amendments:  
Claims 1-6 filed with the letter of 21 July 1999.

This report has been drawn on the basis of the drawings.  
sheets, NONE, as originally filed.  
sheets, NONE, filed with the demand.  
and additional amendments:  
Sheets 1-15 filed with the letter of 21 July 1999.



## PATENT COOPERATION TREATY

## PCT

## INTERNATIONAL PRELIMINARY EXAMINATION REPORT

(PCT Article 36 and Rule 70)

REC'D 21 SEP 1999

WIPO PCT

Applicant's or agent's file reference GCLL-115PC	FOR FURTHER ACTION See Notification of Transmittal of International Preliminary Examination Report (Form PCT/IPEA/416)	
International application No. PCT/US98/26384	International filing date (day/month/year) 11 DECEMBER 1998	Priority date (day/month/year) 12 DECEMBER 1997
International Patent Classification (IPC) or national classification and IPC IPC(6): B26F 1/38, B28B 1/087, B65D 5/54 and US Cl.: 364/468.04, 83/32, 264/72, 229/212		
Applicant NESTECH INC.		

1. This international preliminary examination report has been prepared by this International Preliminary Examining Authority and is transmitted to the applicant according to Article 36.

2. This REPORT consists of a total of 4 sheets.

☒ This report is also accompanied by ANNEXES, i.e., sheets of the description, claims and/or drawings which have been amended and are the basis for this report and/or sheets containing rectifications made before this Authority. (see Rule 70.16 and Section 607 of the Administrative Instructions under the PCT).

These annexes consist of a total of 61 sheets.

3. This report contains indications relating to the following items:

- I ☒ Basis of the report
- II ☐ Priority
- III ☐ Non-establishment of report with regard to novelty, inventive step or industrial applicability
- IV ☐ Lack of unity of invention
- V ☒ Reasoned statement under Article 35(2) with regard to novelty, inventive step or industrial applicability; citations and explanations supporting such statement
- VI ☐ Certain documents cited
- VII ☐ Certain defects in the international application
- VIII ☐ Certain observations on the international application

Date of submission of the demand 02 JULY 1999	Date of completion of this report 22 JULY 1999
Name and mailing address of the IPEA/US Commissioner of Patents and Trademarks Box PCT Washington, D.C. 20231 Facsimile No. (703) 305-3230	Authorized officer WILLIAM GRANT <i>For William Grant</i> Telephone No. (703) 308-1108

## INTERNATIONAL PRELIMINARY EXAMINATION REPORT

International application No.

PCT/US98/26384

## I. Basis of the report

1. This report has been drawn on the basis of *(Substitute sheets which have been furnished to the receiving Office in response to an invitation under Article 14 are referred to in this report as "originally filed" and are not annexed to the report since they do not contain amendments):*

☐ the international application as originally filed.

☒ the description, pages (See Attached) , as originally filed.

pages \_\_\_\_\_ , filed with the demand.

pages \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.

pages \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.

☒ the claims, Nos. (See Attached) , as originally filed.

Nos. \_\_\_\_\_ , as amended under Article 19.

Nos. \_\_\_\_\_ , filed with the demand.

Nos. \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.

Nos. \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.

☒ the drawings, sheets/~~fig~~ (See Attached) , as originally filed.

sheets/~~fig~~ \_\_\_\_\_ , filed with the demand.

sheets/~~fig~~ \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.

sheets/~~fig~~ \_\_\_\_\_ , filed with the letter of \_\_\_\_\_.

2. The amendments have resulted in the cancellation of:

☒ the description, pages NONE .

☒ the claims, Nos. NONE .

☒ the drawings, sheets/~~fig~~ NONE .

3. ☐ This report has been established as if (some of) the amendments had not been made, since they have been considered to go beyond the disclosure as filed, as indicated in the ~~Supplemental Box~~ Additional observations below (Rule 70.2(c)).

4. Additional observations, if necessary:

NONE

## INTERNATIONAL PRELIMINARY EXAMINATION REPORT

International application No.

PCT/US98/26384

**V. Reasoned statement under Article 35(2) with regard to novelty, inventive step or industrial applicability; citations and explanations supporting such statement****1. STATEMENT**

Novelty (N)

Claims	<u>NONE</u>	YES
Claims	<u>1-6</u>	NO

Inventive Step (IS)

Claims	<u>NONE</u>	YES
Claims	<u>1-6</u>	NO

Industrial Applicability (IA)

Claims	<u>NONE</u>	YES
Claims	<u>1-6</u>	NO

**2. CITATIONS AND EXPLANATIONS**

Claims 1-6 meet the criteria set out in PCT Article 33(2)-(4), because the prior art does not teach or fairly suggest steps (i), (ii) of claim 1, also (i), and (i) one (iii) of claim 3, step E, (iii), (V) of claim 5.

Claims 1-6 meet the criteria set out in PCT Article 33(2)-(4), because the prior art does not teach or fairly suggest steps (i), (ii) of claim 1, also (i), and (i) one (iii) of claim 3, step E, (iii), (V) of claim 5.

**Supplemental Box**

(To be used when the space in any of the preceding boxes is not sufficient)

Continuation of: Boxes I - VIII

Sheet 10

**1. BASIS OF REPORT:**

This report has been drawn on the basis of the description,  
pages, NONE, as originally filed.  
pages, NONE, filed with the demand.  
and additional amendments:  
Pages 1-39 filed with the letter of 21 July 1999.

This report has been drawn on the basis of the claims,  
numbers, NONE, as originally filed.  
numbers, NONE, as amended under Article 19.  
numbers, NONE, filed with the demand.  
and additional amendments:  
Claims 1-6 filed with the letter of 21 July 1999.

This report has been drawn on the basis of the drawings,  
sheets, NONE, as originally filed.  
sheets, NONE, filed with the demand.  
and additional amendments:  
Sheets 1-15 filed with the letter of 21 July 1999.

## METHOD AND SYSTEMS FOR NESTING OBJECTS

5 The present invention is in the field of manufacturing, processing and design,  
and also packaging. Nesting is the process of placing objects together with minimal  
spaces between them. The invention is described below in two forms. The first  
form, set forth in Part I below, is directed to methods and systems for nesting of two  
dimensional (2D) objects in a target plane. The second form, set forth in Part II  
below, is directed to methods and systems for nesting three dimensional (3D) objects  
in a target volume. The specific details of the preferred embodiments are set forth in  
10 those respective parts.

# **I. METHOD AND SYSTEMS FOR NESTING TWO DIMENSIONAL OBJECTS IN A TARGET PLANE**

## **FIELD OF THE INVENTION**

5           The present invention is in the field of manufacturing processing and more particularly relates to methods and systems for nesting of two dimensional objects. Nesting is the process of placing objects together with minimal space between them.

## **BACKGROUND OF THE INVENTION**

10           There are many design and manufacturing processes in which substantially two dimensional parts-to-be-assembled are initially laid out on and then cut from planar or sheet stock. For example, leather shoes, clothing, pocketbooks, and automobile and furniture upholstery are often formed from a plurality of panels, or parts, cut from a cowhide. After cutting of the panels for such an article,  
15           conventional seam joining techniques, such as sewing, are used to assemble the panels to form the finished article. Clothing, upholstery and many other articles of other materials are similarly designed and manufactured in a similar manner. Such other articles include body parts for automobiles, for example, hoods or fenders, which may be stamped from metal sheets, and then subsequently shaped to form the  
20           final part configuration. Numerous other articles are similarly manufactured from planar sheets as well.

          In the manufacture of all of these parts, it is important to minimize the waste which remains after the parts are cut from the sheet stock. While in some cases, where the waste can be re-used, such as in the case of sheet metal, this factor is of  
25           lesser importance, there are many cases where the minimization of waste is of critical importance to the commercial viability of a product. The latter cases particularly include shoe manufacture, where the cost of materials, for example cowhides or synthetic materials, is very high.

          In view of such cost considerations, manufacturing processes generally  
30           include attempts to minimize waste. However, the conventional attempts at such minimization have been relatively ineffective. In the leather shoe industry, for example, parts are often manually die cut from cowhides, where the die is manually

placed on the hide by skilled (or relatively unskilled) workers, typically resulting in maximum yields in the 50-60% range for the typical irregularly shaped parts. In recent years, computer controlled layout of parts on a hide have been achieved, resulting in typical yields as high as the 60-70% range.

5 In U.S. Patent No. 4,210,041, the present inventors disclosed a method for laying out, or nesting, a plurality of identical irregular non-polygonal rounded pieces on a parallel edge-bounded sheet material with relatively little waste. In accordance with the method of that patent, the salient points of the part boundary are connected by straight lines to box the part boundary in a plane polygonal figure. Then, pairs of  
10 such plane polygonal figures are pairwise positioned to within a plane closed parallelogram. Finally, a plurality of such parallelogram are arranged side-by-side to define a multiple repeat pattern of objects equal in width to the planar sheet from which the parts are to be cut. A nest constructed in accordance with U.S. Patent 4,210,041 does offer improvement in nesting over prior art techniques; however, the  
15 resulting nests are still less than optimal.

It is an object of the present invention to provide an improved method and system for defining a nest of two dimensional objects.

It is another object to provide an improved method and apparatus for cutting a plurality of two dimensional parts from a sheet with high yield.

20

## **SUMMARY OF THE INVENTION**

The invention is a method and system for defining placement of a plurality of two-dimensional objects on a target planar surface with minimal void space between them. The invention automatically packs the maximum number of irregular parts  
25 possible (preferably, of up to two different shapes) with minimum wasted surface area. The invention uses a heuristical approach, so that intelligent decisions are made in order to carry only a small percentage of intermediate solutions to completion. In typical cases, this enables the computation time used in the nesting process to be reduced from hours to minutes, allowing for a more interactive nesting design  
30 environment.

The invention uses a combination of topology analysis and numerical iteration to determine an efficient layout in short computation times. The process

commences by defining a first sub-nest in a central part of the target, grouping parts to each other, and then "grows" or extends outward toward the boundaries of the target. The resultant layouts are generally parallelogramic and thus angled in nature. The "angled" layout is contrary to conventional rectangular (90 degree) nesting practice. A heuristic approach is used with an interactive process to curtail long computation times and to produce nested layouts much faster.

The nesting method and system of the invention maximizes material utilization from the "center outwards" by minimizing the voids between nested parts. In contrast, conventional nesting practice first defines a target area and then fits parts starting from one edge and "flowing" parts across the target area until the opposite edge is reached.

The heuristics used to curtail long computation times, are intermediate measures of how good a nest is. As the process proceeds from one stage to the next, measures of nesting effectiveness are calculated and only a small percentage of all the possibilities are allowed to continue to higher stages. This "nest culling" heuristical process is effective in substantially reducing computation times. By allowing only the best intermediate nests to proceed to the "tiling" stages, layout times are typically reduced with no practical loss in nesting efficiency.

## BRIEF DESCRIPTION OF THE DRAWINGS

The foregoing and other object of this invention, the various features thereof, as well as the invention itself, may be more fully understood from the following description, when read together with the accompanying drawings in which:

FIG. 1 shows a block diagram of a system in accordance with the invention;

FIG. 2 shows an exemplary part to be nested;

FIG. 3 shows the total nest extent for a nest of the part of FIG. 2;

FIG. 4 shows the partial nest extent for a nest of the part of FIG. 2;

FIGS. 5A and 5B shows two nest approaches for a nest of the part of FIG. 2;

FIGS. 6A-6E show the nesting procedure for the exemplary part of FIG. 2;

FIG. 7 shows a set up and actual tile for a primary tile for nests of the exemplary part of FIG. 2;

FIG. 8 illustrates the secondary tiling step of the invention; and



FIGS. 9 and 10 show exemplary nesting layouts determined using the method and system of the invention.

#### DESCRIPTION OF THE PREFERRED EMBODIMENT

5           A system 10 embodying the invention is shown in Fig. 1. The system 10 includes a part input device 12 a programmed digital computer 14 with an associated display 16 and an output layout buffer printer 18. The part input device 12 may be a conventional computer running a computer aided design program, such as AutoCad. The computer may be an IBM type Pentium-based or an Apple PowerPC-based  
10       personal computer (with a conventional monitor for display 16) programmed to perform the process of the invention. The layout buffer 18 may include a conventional storage medium for storing the layouts determined by the programmed digital computer 12.

          The following description sets forth how two dimensional (2D) parts are  
15       input and defined and how their areas are computed. Data structures are then defined that hold the nesting and tiling information as well as the measure for their incremental effectiveness (permitting use of the heuristics) along the process. Finally, the following sequential steps in the procedure: (1) Nesting, (2) Primary Tiling, (3) Secondary Tiling, and (4) Layout.

20           The computer 12 uses a program that compresses 2D irregular figures (from device 12) onto a target stock area, with minimum area wasted between the figures. The program combines geometric analysis of the parts to be laid out and numerical iteration for exhaustiveness. As a main feature of the process, the parts are nested into parallelogramic orientations and not rectangular orientations (the traditional method).  
25       This flexibility in orientation allow a larger compression of the parts and a lesser waste of material stock. In the preferred embodiment this computer program nests only two different geometrical shapes, and then "tiles" those nested pairs. The program also performs "nest culling" to avoid unnecessary computation. Major tasks performed in the invention are:

30           Task 1:       Define and implement input of the geometry of parts (components) and the target.

- Task 2: Provide an intersection algorithm/process.
- Task 3: Add multiple numerical iteration loops for the various nesting/tiling steps.
- 5 Task 4: Implement measurement of nesting success as void areas and nest extent areas.

**Task 1:**

10 The component parts (to be nested) and the target (upon which the parts are nested) are defined in two dimensional space. In this illustrated embodiment, a part can be "drawn" in the two dimensional computer screen of monitor 16 and the computer 14 then stores this information as a list of x-y coordinates for the vertices of the boundary of a piecewise linear model of the part. In other words, the parts are

15 defined as multisided polygons. These polygons can typically have 10's or 100's of sides depending on the complexity of the figure (curvature of its sides) and the required accuracy of the nesting solution. As a result, parts are stored in files which consist of a 2 column table of x,y coordinate pairs for all the vertices of the part.

20 **Task 2:**

The intersection procedure checks for collisions between parts which may be considered to translate in two dimensional space. This is done in two dimensions by computing the smallest distance between all the vertices of a part number 1 ( $x_1, y_1$ ) and a part number 2 ( $x_2, y_2$ ). This is  $((x_2^2 - x_1^2) + (y_2^2 - y_1^2))^{1/2}$ . In addition, this check for

25 collisions is done between every "peripheral node" of component 1 with every "peripheral node" of part 2. Once that smallest distance is obtained, it is subtracted from every node on one of the parts into what constitutes the translation of this part toward the other one until the two parts basically "touch" each other in the two dimensional space.

30 The intersection procedure constitutes a translation in a particular direction which is not controlled by the user, but is determined by the way the parts are placed in the "working area". In order to control the directionality of the

"approach/collision" process, the intersection of part 1 vertices with part 2 sides and part 2 vertices with part 1 sides is determined. The computation of vertex-to-side distances is somewhat more involved and is described below in connection with the nesting section of the process. In addition "minimum distance part-to-part approaches" are done in the X (horizontal) direction. Directionality is achieved by rotating the parts, nests or tiles a specified angle.

### Task 3:

The nesting and tiling steps have "primary" and "secondary" moves. These moves are made so as to bring two parts or two nests close to each other in order to package them as tightly as possible. The primary move is made in the X direction. The parts (or nests) are placed side by side and then one of them is offset in the Y direction. The amount of this offset (from a maximum of the part's height to the negative of the part's height) constitutes the first iteration loop. In other words, the primary move is made for every Y offset. Once the primary move is made, the secondary move is performed in the Y direction. Then a rotation of the figures is performed for the appropriate angle (90 degrees in this case) so as to bring the approaching direction in the X direction. Then, the secondary move is performed again in the X direction. This primary/secondary move iteration loop is nested within a part side-to-side combination loop. This side-side- combination loop cycles through all the possible side-side combinations of the parts nested. In addition, after every X direction move (for both primary and secondary approaches), a "tangential" move is made in the direction of the intersecting vertex/side. Again the system is rotated by the appropriate angle and another X direction approach is performed. (This slides the vertex along the intersecting side for further compactness). Finally, within the primary/secondary and tangential moves, another loop is performed which cycles through all possible vertex-to-side combinations while looking for the minimum approach intersection process set forth under Task 3 above.

### Task 4:

The necessary geometrical computations for the size of the nest are "Total Event" are determined in Task 4. Obtaining the area of an assembly of components

is a straight forward calculation. The extent of the void areas between the components is determined. Typically, this is a relatively complex and computer intensive calculation. However, instead of computing nest's void areas, the preferred form of the invention calculates the "Partial Extent" of a nest. As set forth below, in the nesting section of the process description, this partial extent includes a measure of the area intersected between parts when forming a nest. This area saved is the inverse or complement of the void area proposed as a heuristic. This "intersection" saved area is much easier to calculate, and has similar bearing in terms of the effectiveness of a nest as the proposed wasted void area.

The success of the heuristic-based process for nest culling is evaluated, along with total extent and partial extent. These heuristics limit the number of nests passed to the primary tiling stage, and are also applied to the primary tiling passage to secondary tiling stage. This further limits the search tree and has dramatically improved the performance of the expensive (computer time-wise) tiling stage. A system of weighting factors is used for the two main heuristic variables, so as to ascertain the relative importance of one versus the other.

#### COMPUTER PROGRAM DESCRIPTION

This section provides a brief description of the program used in computer 14. The hardware platform is an Apple Power Macintosh computer with a 17" color monitor and laser printer. The CAD software used for this embodiment consists of two packages - Autodesk's AutoCAD and Graphsoft's MiniCAD. This embodiment was developed using the following combination of software tools: 1 - Macintosh Programmer's Workshop (MPW) for the development and coding environment (including editing and debugging); 2 - FORTRAN Compiler for the compilation of the program; and 3 - MiniCAD for the part input and output layout visualization. The CAD package allows ease in description on the parts, and interactive visualization of the layouts. Alternatively, AutoCAD could be used

The inner regions of the nests have great importance in the invention (as opposed to the peripheral or edge regions near the target's boundaries.) The computation is segmented into four sequential steps: Nesting, Primary Tiling, Secondary Tiling, and Layout. The process is initiated with the nesting step, where

two irregular objects or parts are taken and brought together until they "touch" each other on their periphery. This grouping of two parts is called a "nest". The parts are brought towards each other as close together as possible for form as compact as possible a nest. Depending on the complexity of the parts being nested, the computer  
5 program actually performs from many hundreds to many thousand different nests and ranks them according to their compactness (using heuristic variables previously described).

Once the nests are formed and ranked, a predetermined number of these nests (the ones that ranked highest) are passed to the primary tiling step. In this step two  
10 nests are brought together in the X direction to form a primary tile or "ptile". A ptile is made of two nests or four parts. The same rational is used to form a ptile as to form a nest. In other words, a grouping of two nests is created that forms the most compact unit. Typically, a few hundred ptiles are generated per each nest passed from the nesting step. These ptiles are consequently ranked using heuristic variables  
15 for compactness.

A small percentage of the ptiles formed (a predetermined number) are then passed onto the secondary tiling step. In this case, two ptiles are brought together in the Y direction to form a secondary tile or "stile". A stile is made up of two stiles or four nests or eight parts. Again, the same rational as before for nest formation is  
20 used, and in the process, over a thousand different stiles are generated for each of the ptiles passed onto this step from the primary tiling step. Finally, a simple layout routine takes the data generated for the best secondary tile and computes the ideal parallelogramic area needed to contain a single nest. A computation of the utilization ratio follows by calculating the actual part area and dividing that by the needed  
25 parallelogramic area. Then the layout routine creates a file with the CAD commands that generate the best layout for the parts nested.

A step by step description of the computer program will now be described, starting with a description of the parts and ending with a description of the computation steps: Nesting, Tiling and Layout.

### Part Description

In order to describe and input parts in two dimensions, they are defined as

polygons in the xy plane. Each polygon can be made of (typically) 100 to 1000 vertices described by x,y coordinate pairs. This method for part input enables description of parts in a simple way, which include curvatures, for the which small polygonal sides are used (vertices are very close together where the curvatures need to be described accurately).

As an example of this methodology, Fig. 2 shows an exemplary part, composed of ten vertices in the xy plane and labeled with coordinates  $(x_1, x_2)$  through  $(x_{10}, y_{10})$ . This simple figure is thus fully described with a set of 10 x,y coordinates pairs for a total of twenty numbers. The part description for the part of Fig. 2 is stored in a small text file with all the x,y coordinate pairs. Thus, design and input of parts is very simple using this mode.

### Part Area Calculation

Part areas are calculated as three different areas: (1) the actual part area, (2) the total area extent for nests and tiles, and (3) the partial area extent for nests and tiles. The computation of the part area is straight forward. If  $y_{\min}$  is the smallest y coordinate, then the part area A is given by:

$$A = \sum_{i=0}^{N_v-1} \frac{1}{2}(X_{i+1} - X_i)(Y_{i+1} + Y_i - 2Y_{\min})$$

where  $N_v$  is the number of vertices, ( $N_v = 10$  in the case of the part of Fig. 2).

The total extent area is calculated as the smallest rectangle that can contain a particular nest or tile. This is illustrated in Fig. 3. This total area extent only involves the knowledge of the maximum and minimum x,y values for the parts within the nest or tile. The partial area extent illustrated in Fig. 4, is a more accurate measure of the actual area used by the individual parts within the nest or tile.

This partial extent is calculated by first computing the areas of the smallest rectangles encompassing individual parts and subtracting the "intersecting" area between the parts being grouped into the nest or tile. It is a smaller area than the total extent and it points more directly to a measure of part grouping compactness.

The two extents described above are used through out the various steps in the computer program as the two heuristic variables to determine the effectiveness of a particular nest or primary tile. Weighting factors are assigned as inputs to the computer program for these two extents so as to prefer one more to the other in the calculation. For the illustrated embodiment, both weights are 1.0 (but other values may be used) for nest and primary tile culling.

As an example of how these extents are computed, the following algorithms show these calculations for a nest made up of left and right parts (subscripts "l" and "r"). Here, min,max corresponds to the minimum, maximum coordinate a given part:

10

$$TotExt = [\max(x_{max,l}, x_{max,r}) - \min(x_{min,l}, x_{min,r})] * [\max(y_{max,l}, y_{max,r}) - \min(y_{min,l}, y_{min,r})]$$

$$Ext_l = (x_{max,l} - x_{min,l}) * (y_{max,l} - y_{min,l})$$

$$Ext_r = (x_{max,r} - x_{min,r}) * (y_{max,r} - y_{min,r})$$

15

$$PartExt = Ext_l + Ext_r - [x_{max,l} - x_{min,r}] * [\min(y_{max,l}, y_{max,r}) - \max(y_{min,l}, y_{min,r})]$$

### Basic Structures

The computer program contains five storage structures, where all the data for the various characteristics of the parts, nests, tiles and layouts are accumulated. They are the /Part/, /Nest/, /Ptile/, /STile/ structures (denoted symbolically surrounded by slashes "/"). These variables are more completely described below. For reference, the variables within each structure are now briefly described:

#### /Part/

25

/Part/ contains variables for the storage of the following part characteristics:

1. Part name which correspond to the text file which stores the part's data
2. Number of vertices -  $N_v$
3. The x-y coordinates for the minimum and maximum values-  $X_{min}$  ...  
 $Y_{max}$
- 30 4. The X-Y arrays for the x-y coordinates of dimension  $N_v$

**/Nest/**

/Nest/ contains variables for the storage of the following nest characteristics, each of which is dimensioned to (100) for the maximum number of possible nest that could be generated:

- 5           1.      Left,Right part's side number - nsidel, nsider (an integer between 1-4)
2.      The right part's displacement relative to the left part to form the nest -  
                  dxn,dyn
3.      The total and partial extent for the nest - TotExtn,PartExten
4.      The ranked sequence from best to worst nest - Seqncn
- 10          5.      The total number of nests generated by the nesting step (not an array) -  
                  Nn

**/PTile/**

15           /PTile/ contains variables for storage of the following Primary Tile characteristics, each of which is dimensioned to (50,10) for the maximum number of possible nests that are allowed to pass to the primary tiling step = 50, and the maximum number of ptils generated per each nest passed = 10.

1.      The right nest's displacement relative to the left nest, to  
                  form the primary tile - dxp,dyp
- 20          2.      The total and partial extent for the ptile - TotExtp,PartExtp
3.      The ranked sequence from best to worst ptile for both  
                  the nest passed and the ptile for that nest (singly  
                  dimensioned to (50) - Nseqncp,TSeqncp
- 25          4.      The total number of ptils generated by the primary  
                  tiling step for each nest passed to it (singly  
                  dimensioned to (50)) - Np

**/Stile/**

30           /Stile/ contains variables for the storage of the following Secondary Tile characteristics, each of which is dimensioned to (50,1080) for the maximum number of possible ptils that are allowed to pass to the secondary tiling step = 50, and the maximum number of stiles generated per each ptile passed = 1080.



1. The top ptile's displacement relative to the bottom ptile, to form the secondary tile dxs,dys
2. The effective parallelogram area used per part - Pa
3. The ranked sequence from best to worst stile for both the ptile passed and the stile for that ptile(singly dimensioned to (50)) - Nseqncs,TSeqncs
4. The total number of stiles generated by the secondary tiling step for each ptile passed to it (singly dimensioned to (50)) - Ns

#### **/Layout/**

/Layout/ contains variables for the storage of the following Layout characteristics.

1. The nest displacement for the best layout - dxlayn,dylayn
2. The ptile displacement for the best layout - dxlayp,dylayp
3. The stile displacement for the best layout - dxlays,dylays
4. The smallest effective parallelogram area used per part, in the final layout - Pamin

#### **Nesting**

Nesting is the process by which two irregular objects or parts are taken and brought together until they "touch" each other on their periphery. This grouping of two parts is called a "nest". A nest is typically formed from two different parts, but for the pictorial examples that follow, the same part is used. The parts are brought towards each other as close together as possible to form a nest as compact as possible.

First, the parts to be nested are circumscribed inside the smallest possible rectangle. The four lateral faces of this rectangle are numbered from 1 to 4. A loop is set up to go through all possible combinations for each face of the left part to be approached by each face of the right part, a total of the combinations in all. Figures 5A and 5B illustrate two nest approaches where Fig. 5A shows nesting for sides 1

and 2 (left and right part respectively) and Fig. 5B shows sides 1 and 1.

Nesting is accomplished in two steps: the axial move (in the x direction or direction of approach), and the tangential move for further reduction of the overall area of the nest. The left part of the nest remains static whereas the right part is  
5 iteratively moved towards the left part in search for the maximum approach to it. Since the approach is in the x direction, a grid is first defined in the y direction which contains the starting locations for the right part's approach toward the left part. A total of ten grid locations is implemented in the y direction. Combining these y grid points with the number of face to face combinations, the maximum number of nests  
10 is generated by the program, which is 100.

The axial move is accomplished by calculating  $N_v$  intersection distances from the left part's vertices towards the right part's sides, and  $N_v$  intersection distances from the right part's vertices toward the left part's sides. The amount that the right part can now be moved towards the left one is the smallest of the  $2-N_v$  distances  
15 calculated for each of the 100 (maximum) nests to be computed.

The tangential move is an adjustment that is implemented once the axial move has been performed. For the tangential move, the part displaced with the intersecting vertex along the other part's intersecting side in the direction along this side until another collision occurs forming a more compact nest. In order to  
20 accomplish this move, the parts are rotated by an angle such that the intersecting side is horizontal, allowing use of the same intersection routine as for the axial move, saving a great deal of computation time.

Figs. 6A-6E show this nesting procedure for the exemplary part of Fig. 2. Fig 6A shows the set up before the axial move. Fig. 6B shows the geometry after the  
25 axial move. Fig. 6C shows the situation after rotation is done so as to render the tangential direction horizontal. At this point, a similar intersection algorithm procedure to the axial move is performed in order to obtain the smallest intersecting line to further approach the right part towards the left one. Fig. 6D depicts the set up after the right part has been moved, and finally; Fig. 6E shows the final nested  
30 situation with the frame of reference rotated back to the original position.

Once the nests are formed, their total and partial area extents are calculated and stored appropriately. Then, these nests are sorted and sequenced, from best to

worst, by seeking for the smallest area extents. Weight factors are applied to each of the two heuristic variables in order to investigate their individual effectiveness. This process is referred to as Nest Ranking.

## 5      **Primary Tiling**

Once the nests are formed and ranked, a predetermined number of these nests (the ones that ranked highest) are passed to the primary tiling step. In this step, two nests are brought together in the x direction to form a primary tile or "ptile". A ptile is made of two nests or four parts. The same rationale is used to form a ptile as to form a nest. The primary tiling procedure starts by placing two nests side by side in the x direction. As in the nesting step, the left nest remains static whereas the right nest is moved towards the left one as much as possible.

A ten point grid is formed in the y direction for the iterative. Since the maximum number of nests passed to the primary tiling step is 50, a maximum of 500 ptilers are generated. Once the two nests are set up and made ready for axial compression (in the x direction), the intersection algorithm is recalled in order to calculate the smallest distance to approach the right nest towards the left one until any part on the right nest touches any part on the left nest. The parts set up in this process is done in the following manner:

- 20                      Part 1: Left Part - Left Nest
- Part 2: Right Part - Left Nest
- Part 3: Left Part - Right Nest
- Part 4: Right Part - Right Nest

25                      The intersection algorithm routine is then performed four times in sequence to check for the smallest axial approach. The routine is called for parts 1 and 3, then for parts 1 and 4, then for parts 2 and 3, and finally for parts 2 and 4. This allows investigating of the collision of any of the parts on the left nest with any of the parts on the right nest. Fig. 7 shows first the set up situation for a particular primary tile of the exemplary part and then the actual tile achieved.

## Secondary Tiling

A small percentage of the ptiles formed (a predetermined number) are then passed onto the secondary tiling step. In the secondary tiling step, as shown in Fig. 8, two ptiles are brought together in the y direction to form a secondary tile or "stile".

5 An stile is made up of two ptiles or four nests or eight parts. In order to set up the geometry for secondary tiling, first two ptiles are placed on the bottom of the domain. Then one more nest is set up along the top that will be nested downwards towards the two ptiles near the bottom of the space. The two ptiles on the bottom are composed of parts numbered 1 through 8 from left to right. The two parts making up the nest at  
 10 the top are labeled 9 and 10. The top nest (the one to be dynamically moved to form the stile) is positioned according to whether the ptile has downward or upward flow ( $dyp < 0$  or  $dyp > 0$ ).

Fig. 8 describes the geometry prior to the secondary tiling iteration for the case of  $dyp > 0$ . In that Figure the top nest is positioned just above the bottom left  
 15 three nests and to the left of the bottom rightmost nest. The top nest is then iterated on a x grid in a similar manner as the ptile process, except that the x displacement for this grid (as shown in the figure) is equal to the primary tile period. Again, the same rationale as before is used and in the process, ten different stiles are generated for each of the ptiles passed onto this step from the primary tiling step.

20 In order to do a y direction secondary tiling, the coordinate system is first rotated 90 degrees and the previously described axial (x direction) intersection algorithm is performed. In this case, the smallest approach distance between any of the two top parts (parts no. 9 and 10) and any of the six bottom parts is detected (parts no. 1 through 6 for the  $dyp > 0$  case). In the case of  $dyp < 0$ , the top nest is set up  
 25 just to the right of the bottom leftmost nest. As a result, the intersection test is made between parts 9 and 10 (top nest) and parts 3 through 8 (bottom tiles).

## Layout

30 Once the secondary tiling is performed, two numbers are known: the primary and the secondary tiling period. These are the numbers that represent the displacements in the primary and secondary tiling directions, which enable replication of as many nests as desired on a skewed plane formation for these parts.

Finally, a simple layout routine takes the data generated for the best secondary tile and computes the ideal parallelogram area needed to contain a single nest. This parallelogram area is calculated using the coordinates for the two displacements necessary to form a layout: the primary tiling period ( $x_p, y_p$ ), and the secondary tiling period ( $x_s, y_s$ ). The area of the parallelogram forms is then calculated from the following formulae:

$$\ell_p = \sqrt{x_p^2 + y_p^2}$$

$$\ell_s = \sqrt{x_s^2 + y_s^2}$$

$$\cos \theta_{ps} = \frac{x_p x_s + y_p y_s}{\ell_p \ell_s}$$

$$A = \ell_p \ell_s \sin \theta_{ps}$$

A computation of the utilization ratio follows by calculating the actual part area and dividing that by the needed parallelogram area. Then, the layout routine creates a file with the CAD commands that generate the best layout for the parts nested.

With the invention, very efficient layouts can be determined with reasonable computation times, using two heuristical variables to eliminate unnecessary branches in the computation process. For simple parts, around 20 nests and primary tiles can be used for passage to higher tiling stages, and still achieve near optimal packing efficiencies. Other numbers of nests and primary tiles can also be used.

Figs. 9 and 10 show exemplary nesting layouts for components of a shoe, which were determined using the invention.

The invention may be embodied in other specific forms without departing from the spirit or essential characteristics thereof. The present embodiments are

therefore to be considered in all respects as illustrative and not restrictive, the scope of the invention being indicated by the appended claims rather than by the foregoing description, and all changes which come within the meaning and range of equivalency of the claims are therefore intended to be embraced therein.

5

## **II. METHOD AND SYSTEM FOR NESTING THREE DIMENSIONAL OBJECTS IN A TARGET VOLUME**

### **REFERENCE TO RELATED APPLICATION**

10           This application is a non-provisional application based on provisional U.S. application Serial No. 60/069,561, filed on December 12, 1997. That application is incorporated by reference herein.

### **FIELD OF THE INVENTION**

15           The present invention is in the field of manufacturing packaging and design and more particularly relates to methods and systems for nesting (or packaging) of three dimensional objects in a target volume. Nesting is the process of placing objects together with minimal space between them.

### **BACKGROUND OF THE INVENTION**

20           There are many design and manufacturing processes in which substantially two dimensional parts-to-be-assembled are initially laid out on and then cut from planes or sheet stock. For example, leather shoes, clothing, pocketbooks, and automobile and furniture upholstery are often formed from a plurality of panels, or  
25           parts, cut from a cowhide. After cutting of the panels for such an article, conventional seam joining techniques, such as sewing, are used to assemble the panels to form the finished article. Clothing, upholstery and many other articles of other materials are similarly designed and manufactured in a similar manner. Such other articles include body parts for automobiles, for example, hoods or fenders,  
30           which may be stamped from metal sheets, and then subsequently shaped to form the final part configuration. Numerous other articles are similarly manufactured from planar sheets as well.

In the manufacture of all of these parts, it is important to minimize the waste which remains after the parts are cut from the sheet stock. While in some cases, where the waste can be re-used, such as in the case of sheet metal, this factor is of lesser importance, there are many cases where the minimization of waste is of critical importance to the commercial viability of a product. The latter cases particularly include shoe manufacture, where the cost of materials, for example cowhides or synthetic materials, is very high.

In view of such cost considerations, manufacturing processes generally include attempts to minimize waste. However, the conventional attempts at such minimization have been relatively ineffective. In the leather shoe industry, for example, parts are often manually die cut from cowhides, where the die is manually placed on the hide by skilled (or relatively unskilled) workers, typically resulting in maximum yields in the 50-60% range for the typical irregularly shaped parts. In recent years, computer controlled layout of parts on a hide have been achieved, resulting in typical yields as high as the 60-70% range.

In U.S. Patent No. 4,210,041, the present inventors disclosed a method for laying out, or nesting, a plurality of identical irregular non-polygonal rounded pieces on a parallel edge-bounded sheet material with relatively little waste. In accordance with the method of that patent, the salient points of the part boundary are connected by straight lines to box the part boundary in a plane polygonal figure. Then, pairs of such plane polygonal figures are pairwise positioned to within a plane closed parallelogram. Finally, a plurality of such parallelogram are arranged side-by-side to define a multiple repeat pattern of objects equal in width to the planar sheet from which the parts are to be cut. A nest constructed in accordance with U.S. Patent 4,210,041 does offer improvement in nesting over prior art techniques; however, the resulting nests are still less than optimal.

Part I of the disclosure as set forth above discloses improvements in nesting of two dimensional objects. That application is incorporated by reference herein.

Further, there is a need to nest three dimensional objects in a target volume. Typically three dimensional objects are manually, or by machine, packed in target volumes with less than optimal techniques. It would be advantageous in many industries to use an optimal technique. For example in the container shipping

industry, containers are packed with objects to be shipped using manually determined placement of the object, generally starting from one border of the container and adding objects one at a time until the container is "full". There are of course cases where an optimized process is easily realized, such as where identical cubic objects  
5 are placed in a container having dimensions which are integer multiples of the dimensions of each side of the cubic object. Generally, however, the cubic shape and those container dimension constraints are not present and the conventional packaging methods result in a "filled" container with substantial void regions between the packed objects.

10 It is an object of the present invention to provide an improved method and system for defining a nest of three dimensional objects.

It is another object to provide an improved method and apparatus for packaging a plurality of three dimensional parts in a target volume.

## 15 SUMMARY OF THE INVENTION

The invention is a method and system for defining placement of a plurality of three-dimensional (3D) objects in a target volume with minimal void space between them. The invention automatically packs the maximum number of irregular parts possible (preferably, of up to two different shapes) with minimum wasted interpart  
20 void regions. The invention uses a heuristic approach, so that intelligent decisions are made in order to carry only a small percentage of intermediate solutions to completion. In typical cases, this enables the computation time used in the nesting process to be reduced from hours to minutes, allowing for a more interactive nesting design environment.

25 The invention uses a combination of topology analysis and numerical iteration to determine an efficient layout in short computation times. The process commences by defining a first sub-nest in a central part of the target volume, grouping parts to each other, and then "grows" or extends outward toward the boundaries of the target volume. The resultant packing layouts are generally  
30 parallelogramic, angled in nature. The "angled" layout is contrary to conventional rectangular (90 degree) nesting practice. A heuristic approach is used with an interactive process to curtail long computation times and to produce nested packing



layouts much faster.

The nesting method and system of the invention maximizes target volume utilization from the "center outwards" by minimizing the void regions between nested parts. In contrast, conventional nesting practice first defines a target area, and then fits parts starting from one boundary of the target volume and "flows" parts across the target volume until the opposite edge is reached.

The heuristics used to curtail long computation times, are intermediate measures of how good a nest is. As the process proceeds from one stage to the next, measures of nesting effectiveness are calculated and only a small percentage of all the possibilities are allowed to continue to higher stages. This "nest culling" heuristical process is effective in substantially reducing computation times. By allowing only the best intermediate nests to proceed to primary, secondary and tertiary "tiling" stages, layout times are typically reduced with no practical loss in nesting efficiency.

#### BRIEF DESCRIPTION OF THE DRAWINGS

The foregoing and other object of this invention, the various features thereof, as well as the invention itself, may be more fully understood from the following description, when read together with the accompanying drawings in which:

- Fig. 11 shows a block diagram of a system in accordance with the invention;
- Fig. 12A and 12B show an exemplary part to be nested;
- Fig. 13 shows the total nest extent for a nest of the part of Fig. 12;
- Fig. 14 shows the partial nest extent for a nest of the part of Fig. 12;
- Figs. 15A and 5B shows two nest approaches for a nest of the part of Fig. 12;
- Figs. 16A-16E show the nesting procedure for the part of Fig. 12;
- Figs. 17A and 17B show two "better" nests of the part of Fig. 12;
- Fig. 18A and 18B show a set up and actual tile respectively for a primary tile for nests of the part of Fig. 12;
- Fig. 19 shows the geometry prior to secondary tiling for the part of Fig. 12;
- Fig. 20 illustrates the secondary tiling step for the part of Fig. 12; and
- Figs. 21, 22 and 23 show final 3D layouts for the part of Fig. 12.

## DESCRIPTION OF THE PREFERRED EMBODIMENT

The invention is a method and system of nesting a plurality of irregular three dimensional parts into a target volume. The method (and system) uses a combination  
5 of topology analysis and numerical iteration to find an efficient layout in short computation times. It starts from an interior (and preferable "center") of the target volume, grouping parts to each other and then "grows" or extends outwards toward the boundaries of the target volume. The resultant packaging layouts are  
10 parallelepiped like in nature. This "angled" layout approach is opposite to conventional rectangular (90 degree) nesting practice. A heuristic approach is used in order to curtail long computation times and to rapidly produce nested layouts, giving a designer a much greater interactive responses in his task.

The method and system of the invention maximizes volume utilization from the "center outwards" by minimizing the voids between nested parts.

15 The heuristics used in order to curtail long computation times, are intermediate measures of how good a nest is. As the computation proceeds from one stage to the next, these measures of nesting effectiveness are calculated and only a small percentage of all the possibilities are allowed to continue to higher stages. This  
20 "nest culling" heuristical process substantially reduces computation times. By allowing only the best intermediate nests to proceed to tiling stages, layout times are reduced typically from hours of computation to minutes, with no practical loss in nesting efficiency.

A system 110 embodying the invention is shown in Fig. 11. The system 110 includes a part input device 112 a programmed digital computer 114 with an  
25 associated display 116 and an output layout buffer 118. The part input device 112 may be a conventional computer running a computer aided design program, such as AutoCad. The computer may be an IBM type Pentium-based or an Apple PowerPC-based personal computer (with a conventional monitor for display 116) programmed to perform the process of the invention. The layout buffer 118 may include a  
30 conventional storage medium for storing the layouts determined by the programmed digital computer 112.

The computer program for computer 110 nests many irregular 3D shapes into

a regularly shaped "box". In defining that program, there are four tasks which are listed below:

5 **Objective I:** Three dimensional extension of the nesting algorithms of Part I of this disclosure as set forth above.

**Task 1:** Define and implement input of the geometry of parts (components) and targets boxes in three dimensions.

10 **Task 2:** Extend our main intersection algorithm to three dimensional domain.

**Task 3:** Add three dimensional numerical iteration loops for the part's depth.

15

**Task 4:** Change measurement of nesting success to void volumes and nest extent volume as opposed to areas."

20 An automatic three dimensional packaging method and system computes the geometric layout of the various components within the target "box" in an efficient and relatively fast turnaround time. The method and system accomplishes three dimensional nesting in reasonable times and obtains high levels of packing densities.

**Task 1:**

25 In Task 1, parts or components to be packed and target boxes are defined and input in three dimensional space. Initially, a two dimensional peripheral polygon for the cross-section of the three dimensional object is defined and stored in terms of a list of x, y coordinates for the vertices of the boundary of the part's cross-section. The part's cross-sections are defined as multisided polygons. These polygons can  
 30 typically have 10's or 100's of sides depending on the complexity of the figure (curvature of its sides) and the

required accuracy of the nesting solution. An extrusion coordinate for every  $x, y$  coordinate of this cross-section  $z(x, y)$  defines a large class of solid components, those which have box like appearance.

5 Nevertheless, not all components may be modeled in this fashion. For example, spheres can not be modeled with this simple extrusion method. Consequently, in those cases a more general method is used that handles all component shapes. In this case, a polygon is first defined for the maximum cross-section. Then this cross-section polygon is subdivided into an  $(x, y)$  grid space of approximately ten subdivisions for each (depending on accuracy desired). Finally,  
10 left and right extrusion values are defined for each grid node. This more complicated description of the component is particularly useful for complex parts.

A "Multiple Extrusion" solid molding technique is used. This technique defines a series of cross-sectional polygons in the  $xy$  plane (typically 5 to 20 cross-sections), which are then multiply extruded/connected in the  $z$  direction. Each cross-  
15 sectional polygon can be made of (typically) 10 to 100 vertices described by  $x, y$  coordinate pairs. In addition each cross-section is associated with a given  $z$  coordinate level or value. After connecting corresponding vertices across " $z$  cross-section levels" a three-dimensional body fitted mesh is formed for the peripheral surface of the parts. This method for part input enables a description of a wide  
20 variety of 3D solids.

#### Task 2:

In Task 2, the intersection algorithm of the above described two dimensional  
25 form of the invention is extended into three dimensions.

In the three dimensional case of the invention, coordinates of the form (x, y, z) are used so that a check for minimum translational intersection distance is:

$$\sqrt{(x_2^2 - x_1^2) + (y_2^2 - y_1^2) + (z_2^2 - z_1^2)}$$

5            This check may be done between every "peripheral node" of component 1 with every "peripheral node" of part 2. Once that distance is obtained, it is subtracted from every node on one of the parts into what constitutes the translation of that part toward the other one until they effectively touch each other. The procedure calculates the intersection of part 1 vertices with part 2 sides and part 2 vertices with  
10           part 1 side, for each of overlapping cross-section planes in the z direction. The computation of vertex to side distances is described more fully below in connection with the nesting section of the program description.

**Task 3:**

15           The invention uses not only "primary" and "secondary" moves in the nesting and tiling steps, but also a "tertiary" move in the component depth direction. A third nested loop is used for numerical iteration in order to apply the three dimensional intersection algorithm in the "z" direction. This third loop is used at every step of the calculational procedure Nesting, Primary Tiling and Secondary Tiling. In addition a  
20           tertiary tiling step is used in order to place nests and ptils into the z direction. This step is described more fully below in connection with the computer program description.

**Task 4:**

25           Geometrical computations are used to determine the size of the nest volume "Total Volume Extent". Obtaining the volume of an assembly of components is a straight forward calculation. The volumetric extent of the void spaces between the components may be computed. However, in the preferred form, Instead of computing nest's void volumes, the "Partial Volume Extent" of a nest is calculated.  
30           As described in more detail, in the nesting section of the computer program

description, this partial extent includes a measure of the volume intersected between parts when forming a nest. This volume saved is the inverse or complement of the void volume used as a heuristic. This "intersection" saved volume is much easier to calculate, and has similar bearing in terms of the effectiveness of a nest as the wasted void volume.

Heuristic algorithms are used for nest culling, total volume extent and partial volume extent. These heuristics limit the number of nests passed to the primary tiling stage and the passage of the primary tiling to second tiling stage. This further limits the search tree and has dramatically improved the performance of the expensive (computer timewise) tiling stage. Many different values for the number of nesting and tiling steps kept and weighting factors may be used for the main heuristic variables. The weighting factors for both the heuristical variables (total and partial volume extents) are constant and set to 1.0, as an example. Other numbers might be used.

#### COMPUTER PROGRAM DESCRIPTION

This section provides a brief description of the computer program used in computer 114. The hardware platform is an Apple Macintosh computer with a 17" color monitor and laser printer. The CAD software used for this embodiment consists of two packages- Autodesk's AutoCAD and Graphsoft's MiniCAD.

This embodiment was developed using the following combination of software tools: 1- Macintosh Programmer's Workshop (MPW) for the operating environment(including editing and debugging); 2- FORTRAN Compiler for the compilation of the program; and 3- MiniCAD for the three-dimensional part input and output layout visualization. The CAD package permits description of the 3D parts using the "Multiple Extrusion" method and interactive visualization of the 3D layouts. AutoCAD's AME solid modeling could alternatively be used to provide a more complete 3D solids description but with a more complex numerical description.

The inner regions of the nest have great importance in the invention (as

opposed to the peripheral or edge regions near the target's boundaries). The computation is segmented into four sequential steps: Nesting, Primary Tiling, Secondary Tiling, and Tertiary Tiling/Layout. The process is initiated with the **nesting** step, two irregular 3D objects or parts are taken and "brought together" until they "touch" each other on their periphery. This grouping of two parts is called a "nest". These two parts are brought towards each other, as close together as possible to form as compact a nest as possible. Depending on the complexity of the parts being nested, the computer program performs from many hundreds to many thousand different nests and ranks them according to the compactness (using heuristic variables as previously described).

Once the nests are formed and ranked, a predetermined number of those nests (the ones that ranked highest) are passed to the **primary tiling** step. In this step, two nests are brought together in the x direction to form a primary tile or "ptile". A ptile is made of two nests or four parts. The same rational is used to form a ptile as to form a nest. That is, a grouping of two nests is created that forms the most possible compact unit. Typically, a few hundred ptils are generated per each nest passed from the nesting step. These ptils are consequently ranked using heuristic variables for compactness.

As small percentage of the ptils formed (a predetermined number) are then passed onto the **secondary tiling step**. In this case, two ptils are brought together in the y direction to form a secondary tile or "stile". A stile is made up of two ptils or four nests or eight parts. Again, the same rational as used for nest formation is used, an in the process, over a thousand different stiles are generated for each of the ptils passed onto this step from the primary tiling step. For each of these secondary tiles produced, a **tertiary tiling** is implemented. In the tertiary tiling secondary tiles are brought together in the z direction, using a simple stacking rational. The stiles are brought only as close as their closest boundaries, without optimization; that step is performed for every stile generated without secondary tile culling. In other embodiments, stiles are nested with appropriate culling to reduce computational time.

Finally, a **layout** routine takes the data generated for the best tertiary tile and

computes the ideal parallelepiped volume needed to contain a single nest. A computation of the utilization ratio follows by calculating the actual part volume and dividing that by the needed parallelepiped volume. Then the layout routine creates a file with the 3D CAD commands that generate the best layout for the parts nested.

5 That layout may be displayed on monitor 116, or stored and/or printed in layout buffer/printer 118.

The following provides a step by step description of the computer program, starting with the method for describing 3D parts and ending with a description of the computation steps: Nesting, Tiling and Layout.

10

### Three Dimensional Part Description

In order to describe and input parts in three dimensions, the program uses a "Solid Modeling" technique called "Multiple Extrusion". Using this technique, a series of cross-sectional polygons are defined in the xy plane (typically 5 to 20 cross-  
15 sections), which are then multiply extruded/connected in the z direction. Each cross-sectional polygon can be made of (typically) 10 to 100 vertices described by x, y coordinate pairs. In addition, each cross-section is associated with a given z coordinate level, or value. After connecting corresponding vertices across "z cross-section levels", a three-dimensional body fitted mesh is formed for the peripheral  
20 surface of the parts. This method for part input enables description of a wide variety of 3D solids including irregular boxes and spherical objects.

As an example of this methodology, Figs. 12A and 12B shows a 3D object  
140 created with four cross-sectional z levels labeled  $z=0, 0.4, 0.8$  and  $1.2$ . These are the values of the z coordinate at each cross-sectional level. Each cross-section in turn  
25 lies on the xy plane and is composed of ten vertices labeled with coordinates  $(x_1, x_2)$  through  $(x_{10}, y_{10})$ . This 3D figure is thus fully described with four sets of ten x, y coordinate pairs for a total of eighty numbers. This particular figure of Figs. 12A and 12B is used in the following illustration of the invention.

The data defining the object 140 is stored in a text file, with all the x, y  
30 coordinate pairs. Thus, the invention enables a relatively straight forward design and input of a large variety of 3D parts using this multiple extrusion model.



## Part Volume Calculation

Three different volumes are calculated: 1- the actual part volume, 2- the total volume extent for nests and tiles, and 3- the partial volume extent for nests and tiles. The computation of the part volume is relatively straightforward: the quadrilateral faces formed by the four vertices across two z levels are defined to be straight two-dimensional planes. If  $dz$  the z coordinate interval between cross-sections is referred to as  $dz$  and  $y_{\min}$  is defined as the smallest y coordinate at each cross-section, then the cross-section areas  $A_k$  and part volume  $V$  are:

$$A_k = \sum_{i=0}^{N_v} \frac{1}{2} (X_{i+1} - X_i) (Y_{i+1} + Y_i - 2Y_{\min})$$

$$V = \sum_{k=1}^{N_z} \frac{1}{2} (A_{k+1} + A_k) dz$$

where  $N_v$  and  $N_z$  are the number of vertices per each cross-section, and the total number of cross-sections, respectively (10 and 4 in the case of object 140).

The total extent volume is the smallest box that can contain a particular nest or tile. This is illustrated on in Fig. 15. This total volume extent is calculated from the knowledge of the maximum and minimum x, y, z values for the parts within the nest or tile. The partial volume extent is illustrated in Fig. 16. That value is an accurate measure of the actual volume used by the individual parts within the nest or tile.

The partial extent is calculated by first computing the volumes of the smallest boxes encompassing individual parts and subtracting the "intersecting" volume between the parts being grouped into the nest or tile. It is a smaller volume than the "total extent" and it points more directly to a measure of part grouping compactness.

The total volume extend and partial extent described above are used throughout the various steps in the computer program as the two heuristic variables to determine the effectiveness of a particular nest or primary tile. Weighting factors are assigned as inputs to the computer program for these two extents so as to prefer one more to the other in the calculation. Both weights are set to 1.0 in the illustrated embodiment, but other values for these heuristic may be used for different nest and primary tile culling.

The algorithms for calculation for a nest made up of left and right parts (subscripts "l" and "r") are shown below. In this example,  $y_{min_l}$  corresponds to the minimum y coordinate for the left part. The total extent (totExt) and partial extent (PartExt) are determined in accordance with:

$$\begin{aligned} \text{TotExt} &= [\max(x_{max_l}, x_{max_r}) - \min(x_{min_l}, x_{min_r})] * \\ &\quad [\max(y_{max_l}, y_{max_r}) - \min(y_{min_l}, y_{min_r})] * \\ &\quad [\max(z_{max_l}, z_{max_r}) - \min(z_{min_l}, z_{max_r})] \\ \text{Ext}_l &= (x_{max_l} - x_{min_l}) * (y_{max_l} - y_{min_l}) * (z_{max_l} - z_{min_l}) \\ \text{Ext}_r &= (x_{max_r} - x_{min_r}) * (y_{max_r} - y_{min_r}) * (z_{max_r} - z_{min_r}) \\ \text{PartExt} &= \text{Ext}_l + \text{Ext}_r - [x_{max_l} - x_{min_r}] * \\ &\quad [\min(y_{max_l}, y_{max_r}) - \max(y_{min_l}, y_{min_r})] * \\ &\quad [\min(z_{max_l}, z_{max_r}) - \max(z_{min_l}, z_{min_r})] \end{aligned}$$

### Basic Structures

The computer program contains five storage structures where all the data for the various characteristics of the parts, nests, tiles and layouts are accumulated. Those structures are the /Part/, /Nest/, /Ptile/, /Stile/ and /Layout/ structures (denoted symbolically surrounded by slashes "/"). These variables are more thoroughly described below. The variables within each structure are:

#### /Part/

/Part/ contains variables for the storage of the following part characteristics:

1. Part name which correspond to the text file which stores the part's vertex data
2. Number of vertices per cross-section layer:  $N_v$
3. Number of cross-section layers:  $N_z$
4. The x, y coordinates for the minimum and maximum values:  $x_{min} \dots y_{max}$
5. The z coordinates for the lowest and highest cross-section layer -  $z_{min}, z_{max}$
6. The z layer interval:  $dz$
7. The X, Y arrays for the x, y coordinates of dimension ( $N_v, N_z$ )

7. The X, Y arrays for the x, y coordinates of dimension ( $N_v$ ,  $N_z$ )
8. The Z array for the z coordinates of dimension ( $N_z$ )

**/Nest/**

5 /Nest/ contains variables for the storage of the following nest characteristics, each of which is dimensioned to (7400) for the maximum number of possible nests that could be generated:

1. Left, Right part's side number - nsidel, nsider (an integer between 1-4)
2. A switch variable indicating whether the right part has been flipped in the z  
10 axis
3. The right part's displacement relative to the left part to form the nest:  $dx_n$ ,  
 $dy_n$ ,  $dz_n$
4. The total and partial extent for the nest: TotExt<sub>n</sub>, PartExt<sub>n</sub>
5. The ranked sequence from best to worst nest: Seqnc<sub>n</sub>
- 15 6. The total number of nests generated by the nesting step (not an array) -  $N_n$

**/PTile/**

/PTile/ It contains variables for the storage of the following Primary Tile characteristics, each of which is dimensioned to (50,710) for the maximum number of possible nests that are allowed to pass to the primary tiling step = 50, and the  
20 maximum number of ptils generated per each nest passed = 710.

1. The right nest's displacement relative to the left nest, to form the primary tile  
-  $dx_p$ ,  $dy_p$ ,  $dz_p$
2. The total and partial extent for the ptil: TotExt<sub>p</sub>, Part Ext<sub>p</sub>
3. The ranked sequence from best to worst ptil for both the nest passed and the  
25 ptil for that nest (singly dimensioned to (50)): NSeqnc<sub>p</sub>, Tseqnc<sub>p</sub>
4. The total number of ptils generated by the primary tiling step for each nest passed to it (singly dimensioned to (50)):  $N_p$

**/STile/**

/STile/ contains variables for the storage of the following Secondary Tile characteristics, each of which is dimensioned to (50,1080) for the maximum number of possible ptils that are allowed to pass to the secondary tiling step = 50, and the  
30 maximum number of stiles generated per each ptil passed = 1080.

1. The top ptile's displacement relative to the bottom ptile, to form the secondary tile:  $dx_s, dy_s, dz_s$
2. The effective parallelepiped volume used per part:  $P_v$
3. The ranked sequence from best to worst stile for both the ptile passed and the  
5 stile for that ptile (singly dimensioned to (50)):  $N_{Seqnc_s}, T_{Seqnc_s}$
4. The total number of stiles generated by the secondary tiling step for each ptile passed to it (singly dimensioned to (50)):  $N_s$

#### /Layout/

10 /Layout/ contains variables for the storage of the following Layout characteristics.

1. The nest displacement for the best layout:  $dxlay_n, dylay_n, dzlay_n$
2. The ptile displacement for the best layout:  $dxlay_p, dylay_p, dzlay_p$
3. The stile displacement for the best layout:  $dxlay_s, dylay_s, dzlay_s$
- 15 4. The z direction displacement for the best layout:  $dxlay_z, dylay_z, dzlay_z$
5. The smallest effective parallelepiped volume used per part, in the final layout:  
 $P_{v_{min}}$

#### Nesting

20 Nesting is the process by which two irregular 3D objects or parts are taken and brought together until they "touch" each other on their periphery. This grouping of two parts is called a "nest". A nest is typically formed from two different parts, but for the pictorial examples that follow the same part is used. The parts are brought towards each other as close together as possible to form a nest as compact as  
25 possible.

In performing this task in three dimensions, the parts to be nested are first circumscribed inside the smallest possible box. In the illustrated embodiment, the bottom and top z surfaces of the bounding box are not fully nested, but only are stacked; in other embodiments, full nesting (optimal) is performed. The other four  
30 lateral faces of this box are numbered from 1 to 4. The program sets up a loop to go through all possible combinations for each face of the left part to be approached by each face of the right part, a total of ten combinations in all. Another ten face-to-face

combinations are to accommodate when the right part is "flipped" along the z axis, yielding a total of twenty face combinations used for nesting. Figs. 15A and 15B illustrate two nest approaches. Fig. 15A shows nesting for sides 1 and 2 (left and right part respectively) and Fig. 15B shows sides 1 and 1 with the right part flipped.

5 Nesting is accomplished in two steps: (1) an axial move (in the x direction or direction of approach), and (2) a tangential move for the further reduction of the overall volume of the nest. In the illustrated embodiment, the left part of the nest remains static while the right part is iteratively moved towards the left part in search for the maximum approach to it. Since the approach is in the x direction, a grid in  
10 the yz plane is first defined that grid contains the starting locations for the right part's approach toward the left part. A total of ten grid locations is implemented in the y direction while in the z direction, in increments of the interlevel distance dz are stepped through.

As a result, the total number of yz plane grid points is  $10(n_{zl} + n_{zr} - 3)$ . Where  
15 the maximum number of z cross-section levels allowed per part is 20, there are a maximum of 370 iterative grid points in the yz plane. The number of nests generated (7400) is determined by combining these yz grid points with the number of face to face combinations. For the exemplary object 140, having only 4 cross-sections there are only 50 yz grid points and a total of 1,000 possible nests generated.

20 The axial move is accomplished by calculating  $N_v$  intersection distances from the left part's vertices towards the right part's sides, and  $N_v$  intersection distances from the right part's vertices toward the left part's sides. This is performed on each and every z level cross-section plane, preferably using the basic 2D intersection algorithm disclosed in the incorporated reference. The intersection algorithm is  
25 executed for every z plane cross-section and for each of the yz plane grid points. The amount that the right part is moved towards the left one is the smallest of the  $2 \cdot N_v \cdot N_z$  distances calculated for each of the 7,400 (maximum) nests to be computed.

The tangential move is an adjustment implemented once the axial move has been performed. For the tangential move, the part is displaced with the intersecting  
30 vertex along the other part's intersecting side in the direction along that side until another "collision" occurs and a more compact nest is formed. In order to

accomplish that move, parts are "rotated" by an angle such that the intersecting side is horizontal, allowing use of the same intersection routine as for the axial move.

5 Figs. 16A-16E show the nesting procedure for one of the z cross-section levels. The first step (Fig 16A) shows the set up before the axial move. The second step (Fig. 16B) shows the geometry after the axial move. The third step (Fig. 16C) shows the geometry after rotation is done so as to render the tangential direction horizontal. At this point, a similar intersection algorithm procedure to the axial move is performed in order to obtain the smallest intersecting line to further approach the right part towards the left one. The fourth step (Fig. 16D) depicts the set up after the right part has been moved, and finally the fifth step (Fig. 16E) shows the final nested situation with the frame of reference rotated back to the original position.

10 Once the nests are formed their total and partial volume extents are calculated and stored. Then, a sorting of these nests is performed and the nests are sequenced from best to worst by seeking for the smallest volume extents. Weight factors are applied to each of the two heuristic variables in order to determine their individual effectiveness. This process is referred to as Nest Ranking. Fig. 17A and 17B show two of the better nests formed with the test object 140.

### Primary Tiling

20 Once the nests are formed and ranked, a predetermined number of the nests (the ones that ranked highest) are passed to the primary tiling step. In this step two nests are brought together in the x direction to form a primary tile or "ptile". A ptile is made of two nests or four parts, and the same rational is used to form a ptile as used to form a nest. The primary tiling procedure starts by placing two nests side by side in the x direction. As in the nesting step, the left nest remains static, while the right nest is moved towards the left one as much as possible.

25 A grid is formed in the yz plane for the iterative approach, with ten grid points in the y direction and a number of z grid points corresponding to all the corresponding intersecting z cross-section levels between the two nests. The maximum number of z grid points  $2(N_{zl} + N_{zr} - 3) - 3$  is 71. Typically, a few hundred ptilines are generated per each nest passed from the nesting step. As a maximum for

the exemplary computer program (only twenty z levels allowed), there are 710 ptils for every nest. For the test object 140, there are only 70 ptils per nest. Since the maximum number of nests passed to the primary tiling step is set to 50, a maximum of 3,550 ptils are generated, but for test object 140, the maximum number of ptils is 350.

Once the two nests are set up and made ready for axial compression (in the x direction), the intersection algorithm is recalled in order to calculate the smallest distance to approach the right nest towards the left one until any part on the right nest touches any part on the left nest. The computer program numbers the parts set up in this process in the following manner:

Part 1: Left Part - Left Nest

Part 2: Right Part - Left Nest

Part 3: Left Part - Right Nest

Part 4: Right Part - Right Nest

The intersection algorithm routine is then performed four times in sequence to check for the smallest axial approach. The routine is called for parts 1 and 3, then for part 1 and 4, then for parts 2 and 3, and finally for parts 2 and 4. This allows determination of the collision of any of the parts on the left nest with any of the parts on the right nest. Figs. 18A and 18B show first the set up situation for a particular primary tile of test object 140 (Fig. 18A), and then the actual tile achieved (Fig. 18B).

### Secondary Tiling

A small percentage of the ptils formed (a predetermined number) are then passed onto the secondary tiling step. In the secondary tiling step, two ptils are brought together in the y direction to form a secondary tile or "stile". An stile is made up of two ptils or four nests, or eight parts. In order to set up the geometry for secondary tiling, first two ptils are placed on the bottom of the domain. Then one more nest is set up along the top that is to be nested downwards towards the two ptils near the bottom of the space. The two ptils on the bottom are composed of parts numbered 1 through 8 from left to right. The two parts making up the nest at the top, are labeled 9 and 10. The top nest (the one to be dynamically

moved to form the stile) is positioned according to whether the ptile has downward or upward flow ( $dy_p < 0$  or  $dy_p > 0$ ).

Fig. 19 shows geometry prior to the secondary tiling iteration for the case of  $dy_p > 0$ . In that figure, the top nest is positioned just above the bottom left three nests and to the left of the bottom rightmost nest. The top nest is then iterated on a xz grid in a similar manner as done in the ptile process, except that the x displacement for this grid (as shown on the figure) is equal to the primary tile period. Again the same rational as before is used, and in the process over a thousand different stiles are generated for each of the ptiles passed onto this step from the primary tiling step.

In order to accomplish a y direction secondary tiling, the coordinate system is rotated 90 degrees, and the previously described axial (x direction) intersection algorithm is performed. In this case a check is performed to determine the smallest approach distance between any of the two top parts (part nos. 9 and 10) and any of the six bottom parts (part nos. 1 through 6 for the  $dy_p > 0$  case). In the case of  $dy_p < 0$  the top nest is set up just to the right of the bottom leftmost nest. As a result, the intersection test is made between parts 9 and 10 (top nest) and parts 3 through 8 (bottom tiles). The resulting stile is shown in Fig. 20.

### **Tertiary Tiling/Layout**

Performance of the primary and secondary tiling steps yields two numbers: the primary and the secondary tiling period. These numbers represent the displacements in the primary and secondary tiling directions, and replication of as many nests as desired on a skewed plane formation for these parts. For each of these secondary tiles produced, a tertiary tiling is also implemented. In the tertiary tiling process of the illustrated embodiment, secondary tiles are brought together in the z direction using a simple stacking rationale, bringing the stiles no closer than their closest boundaries. In that form, the layout is an inexpensive calculation and can be performed for every stile generated without the need for secondary tile culling. In other forms of the invention, the secondary tiled configuration can be optimally nested into stacks with optimal packing. The latter process can be performed with reduced computational time using the above described heuristic method.



The "stacking" of skewed planes on top of each other is performed by displacing the layers to avoid collisions when nests or tiles have been formed with non-zero z direction displacements.

- 5 A simple layout routine takes the data generated for the best tertiary tile and computes the ideal parallelepiped volume needed to contain a single nest. This parallelepiped volume is calculated using the coordinates for the three displacements necessary to form a layout: the primary tiling period ( $x_p, y_p, z_p$ ), the secondary tiling period ( $x_s, y_s, z_s$ ) and the displaced z direction stacking period ( $x_z, y_z, z_z$ ). The volume of the parallelepiped formed is calculated from the following formulae:

$$\begin{aligned} \ell_p &= \sqrt{x_p^2 + y_p^2 + z_p^2} \\ \ell_s &= \sqrt{x_s^2 + y_s^2 + z_s^2} \\ \ell_z &= \sqrt{x_z^2 + y_z^2 + z_z^2} \\ \cos \theta_{ps} &= \frac{x_p x_s + y_p y_s + z_p z_s}{\ell_p \ell_s} \\ \cos \theta_{pz} &= \frac{x_p x_z + y_p y_z + z_p z_z}{\ell_p \ell_z} \\ \cos \theta_{sz} &= \frac{x_s x_z + y_s y_z + z_s z_z}{\ell_s \ell_z} \\ Vol &= \ell_p \ell_s \ell_z \sin \theta_{ps} \sin \theta_{pz} \sin \theta_{sz} \end{aligned}$$

- 10 A computation of the utilization ratio is determined by calculating the actual part volume and dividing that volume by the needed parallelepiped volume. Then the layout routine creates a file with the 3DCAD commands that generate the best layout for the parts nested.

## 15 **Performance of the Illustrated Embodiment**

### **Example 1. Layout Results Vs Number of Nests/PTiles Kept**

- 20 The above defined computer system (Fig. 11) and computer program was implemented for the test object 140 of Fig. 12. The number of nests and ptils kept for passage to further stages of the computation was varied simultaneously between 5

and 50. That is, these two numbers were varied equally ( $N_{n-pt}=N_{pt-st}$ ). A variation from 5 to 50 represents more than two orders of magnitude variation in computation time. The programmed computer produced a layout result for only 5 nests/ptiles kept in less than a minute, whereas it produced a layout for 50 nests/ptile in

5 approximately one hour.

The test object 140 tested is a very simple one with only a few vertices and cross-section levels to describe it. More complex parts may have typically 10 times more vertices and say 5 times more cross-sections to describe it. This fifty-fold increase in computation time means that for a small number of nests/ptiles kept, the computer processing time will now be on the order of an hour, whereas for the large number of nests/ptiles kept, the numerical solution could typically take many hours. Even with the use of faster computers (e.g., a ten fold increase in speed), a long wait would be expected for a 3D layout. The heuristic nest culling process, achieves near optimal layout utilization efficiencies, while using much smaller computation times.

15 The following table shows volume utilization using the configuration of this example, the number of nests/ptiles kept for the test object 140.

	$N_{n-pt}$	$N_{pt-st}$	Percent Volume Utilization
	5	5	35%
20	10	10	41%
	20	20	42%
	50	50	42%

The data in the above table suggests that, keeping 20 nests is all that is needed in order to achieve near optimal results. More complex parts will also require a larger number of nests/ptiles kept for passage onto higher stages in the computation.

The three layouts produced for the three cases of 5, 10 and 20 nests/ptiles kept are shown in Figs. 21-23. The  $N=10$  case (Fig. 22) used a different nest than the  $N=5$  case (Fig. 21) and yielded an increase in efficiency of 6%. The  $N=20$  case (Fig. 23) kept the same nest used in the  $N=10$  case but used a more efficient ptile (which was not kept within the ten limitation) in order to increase the utilization efficiency another 1%. In order to achieve near optimal volume utilization, the weighting

factors of the two heuristic variables can be varied in order to ascertain their relative relevance. Other heuristics can also be used to further shorten computation times, while improving the utilization efficiency.

5 In other embodiments, the computer program incorporates z face nesting and tiling as well as lateral nesting tiling. The tangential move of the nesting step is used in both the primary and second tiling steps to produce even more compact layouts. A full intersecting tertiary tiling step (as opposed to the simple "semi-stacking" tiling procedure) can also be used for improved layouts. A more complex 3D part description may be used, permitting multiple connected cross-sections into the  
10 geometric algorithms of the numerical calculation.

The invention may be embodied in other specific forms without departing from the spirit or essential characteristics thereof. The present embodiments are therefore to be considered in all respects as illustrative and not restrictive, the scope of the invention being indicated by the appended claims rather than by the foregoing  
15 description, and all changes which come within the meaning and range of equivalency of the claims are therefore intended to be embraced therein.

What is claimed is:

- 1 Method defining a packing configuration for a plurality of two dimensional  
(2D) objects in a target plane, comprising the computer generated steps of:
  - 5 A. generating signals defining each of said 2D objects with respect to an object coordinate system having orthogonal axes  $X_0$  and  $Y_0$ , in terms of a plurality of points representative of boundary points on said objects,
  - 10 B. associating two of said objects to define an object pair,
  - C. generating a nest signal representation of said objects of said object pair nested with respect to a nest coordinate system having orthogonal  
15 axes  $X_N$  and  $Y_N$ , to define a plurality of close-packed candidate nests in which the pairwise positions of two said objects of each candidate nest are characterized by relatively small planar spaces between the respective objects,
  - 20 D. selecting a subset of said candidate nests in accordance with predetermined nest criteria,
  - E. for each of said selected candidate nests, generating a tile signal representative of two such nests tiled with respect to a tile coordinate  
25 system having orthogonal axes  $X_T$  and  $Y_T$ , said tiling steps including the sub steps of:
    - I. generating a primary tile signal representative of said  
30 two nests tiled with respect to said  $X_T$  axis by translating said nests with relative motion in the direction of said  $X_T$  and  $Y_T$  axes, to define a plurality of candidate primary tiles in which the pairwise

positions of said two nests of each candidate primary tile are characterized by relatively small planar spaces between said nests,

- 5           ii.     selecting a subset of said candidate primary tiles in accordance with predetermined primary tile criteria,
- 10           iii.    each of said selected primary tiles, generating a secondary tile signal representative of a nest and said primary tile tiled with respect to said  $Y_T$  axis by translating said nest with respect to said primary tile with relative motion in the direction of said  $X_T$  and  $Y_T$  axes, to define a plurality of candidate secondary tiles in which the pairwise positions of said nest and said primary tile are characterized by relatively small planar spaces between said nest and the nests of said primary tile,
- 15           iv.     selecting one of said candidate secondary tiles in accordance with predetermined secondary tile criteria, thereby defining the optimal nest and primary tile, said optimal nest defining said packing configuration.

2.     The method according to claim 1 comprising the further steps of:  
25           F.     generating an initial tile signal representative of an initial secondary tile positioned in said target plane at a position other than adjacent the boundary of said target plane, and  
30           G.     generating an area packing signal representative of said initial tile as augmented additional secondary tile extending successively from said initial secondary tile in the direction of each of said  $X_T$ , and  $Y_T$  axes,

whereby adjacent secondary tiles are substantially identically pairwise configured in said target plane in the direction of said  $X_T$  and  $Y_T$  axes.

3. A system for defining a packing configuration for a plurality of two dimensional (2D) objects in a target plane, comprising:
  - 5 A. means for generating signals defining each of said 2D objects with respect to an object coordinate system having orthogonal axes  $X_0$  and  $Y_0$ , in terms of a plurality of points representative of boundary points on said objects,  
10
  - B. means for associating two of said objects to define an object pair,
  - C. means for generating a nest signal representation of said objects of  
15 said object pair nested with respect to a nest coordinate system having orthogonal axes  $X_N$  and  $Y_N$ , to define a plurality of close-packed candidate nests in which the pairwise positions of two said objects of each candidate nest are characterized by relatively small planar spaces between the respective objects,  
20
  - D. means for selecting a subset of said candidate nests in accordance with predetermined nest criteria,
  - E. tiling means operative for each of said selected candidate nests, for  
25 generating a tile signal representative of two such nests tiled with respect to a tile coordinate system having orthogonal axes  $X_T$  and  $Y_T$ , said tiling means including:
    - 30 i. means for generating a primary tile signal representative of said two nests tiled with respect to said  $X_T$  axis by translating said nests with relative motion in the direction of said  $X_T$  and  $Y_T$  axes, to

define a plurality of candidate primary tiles in which the pairwise positions of said two nests of each candidate primary tile are characterized by relatively small planar spaces between said nests,

5

ii. means for selecting a subset of said candidate primary tiles in accordance with predetermined primary tile criteria,

10

iii. means operative for each of said selected primary tiles, for generating a secondary tile signal representative of a nest and said primary tile tiled with respect to said  $Y_T$  axis by translating said nest with respect to said primary tile with relative motion in the direction of said  $X_T$  and  $Y_T$  axes, to define a plurality of candidate secondary tiles in which the pairwise positions of said nest and said primary tile are characterized by relatively small planar spaces between said nest and the nests of said primary tile,

15

20

iv. means for selecting one of said candidate secondary tiles in accordance with predetermined secondary tile criteria, thereby defining the optimal nest and primary tile, said optimal nest defining said packing configuration.

25

4. The system according to claim 1 further comprising:

F. means for generating an initial tile signal representative of an initial secondary tile positioned in said target plane at a position other than adjacent the boundary of said target plane, and

30

- G. means for generating an area packing signal representative of said initial tile as an augmented additional secondary tile extending successively from said initial secondary tile in the direction of each of said  $X_T$  and  $Y_T$  axes, whereby adjacent secondary tiles are substantially identically pairwise configured in said target plane in the direction of said  $X_T$  and  $Y_T$  axes.
- 5
5. Method of defining a packing configuration for a plurality of three dimensional (3D) objects in a target volume, comprising the computer generated steps of:
- 10
- A. generating an object signal defining each of said 3D objects with respect to an object coordinate system having orthogonal axes  $X_0$ ,  $Y_0$  and  $Z_0$ , in terms of a plurality of points representative of boundary points on said objects,
- 15
- B. associating two of said objects to define an object pair,
- C. generating a nest signal representative of said objects of said object pair nested with respect to a nest coordinate system having orthogonal axes  $X_N$ ,  $Y_N$  and  $Z_N$ , to define a plurality of close-packed candidate nests in which the pairwise positions of said objects of each candidate nest are characterized by relatively small volumetric spaces between the respective objects,
- 20
- 25
- D. selecting a subset of said candidate nests in accordance with predetermined nest criteria,
- E. for each of said selected candidate nests, generating a tile signal representative of two of said nests tiled with respect to a tile coordinate system having orthogonal axes  $X_T$ ,  $Y_T$  and  $Z_T$ , said tile generating step including the sub steps of:
- 30



- 5

i. generating a primary tile signal representative of said two nests tiled with respect to said  $X_T$  axis by translating said nests with relative motion in the direction of said  $X_T$ ,  $Y_T$  and  $Z_T$  axes, to define a plurality of candidate primary tiles in which the pairwise positions of said two nests of each candidate primary tile are characterized by relatively small volumetric spaces between said nests,
- 10

ii. selecting a subset of said candidate primary tiles in accordance with predetermined primary tile criteria,
- 15

iii. for each of said selected primary tiles, generating a secondary tile signal representative of one of said nests of said primary tiles and said primary tile tiled with respect to said  $Y_T$  axis by translating said nest with respect to said primary tile with relative motion in the direction of said  $X_T$ ,  $Y_T$  and  $Z_T$  axes, to define a plurality of candidate secondary tiles in which the pairwise positions of said nest and said primary tile are characterized by relatively small volumetric spaces between said nest and the nests of said primary tile,

20
- 25

iv. selecting one of said candidate secondary tiles in accordance with predetermined secondary tile criteria,
- 30

v. for each of said selected secondary tiles, generating a tertiary tile signal representative of two of said secondary tiles tiled with respect to said  $Z_T$  axis by translating said secondary tiles with respect to each other with relative motion of in the direction of said  $X_T$ ,  $Y_T$ , and  $Z_T$  axes to define a plurality of tertiary tiles

in which the pairwise positions of said secondary tiles  
 are characterized by relatively small volumetric spaces  
 between said secondary tiles,

- 5           vi.     selecting one of said tertiary tiles from said candidate  
                   tertiary tiles in accordance with predetermined tertiary  
                   tile criteria, thereby defining an optimal nest of said  
                   primary tile and said secondary tile, said optimal nest  
                   defining said packing configuration.

10

6.     The method according to claim 5 comprising the further step of
- 15           F.     generating an initial tile signal representative of an  
                   initial tertiary tile positioned in said target volume at a  
                   position other than adjacent the boundary of said target  
                   volume, and

15

- 20           G.     generating a volume packing signal representative of  
                   said initial tile as augmented additional tertiary tiles  
                   extending successively from said initial tertiary tile in  
                   the direction of each of said  $X_T$ ,  $Y_T$ , and  $Z_T$  axes,  
                   whereby adjacent tertiary tiles are substantially  
                   identically pairwise configured in said target volume in  
                   the direction of said  $X_T$ ,  $Y_T$ , and  $Z_T$  axes.

20

25

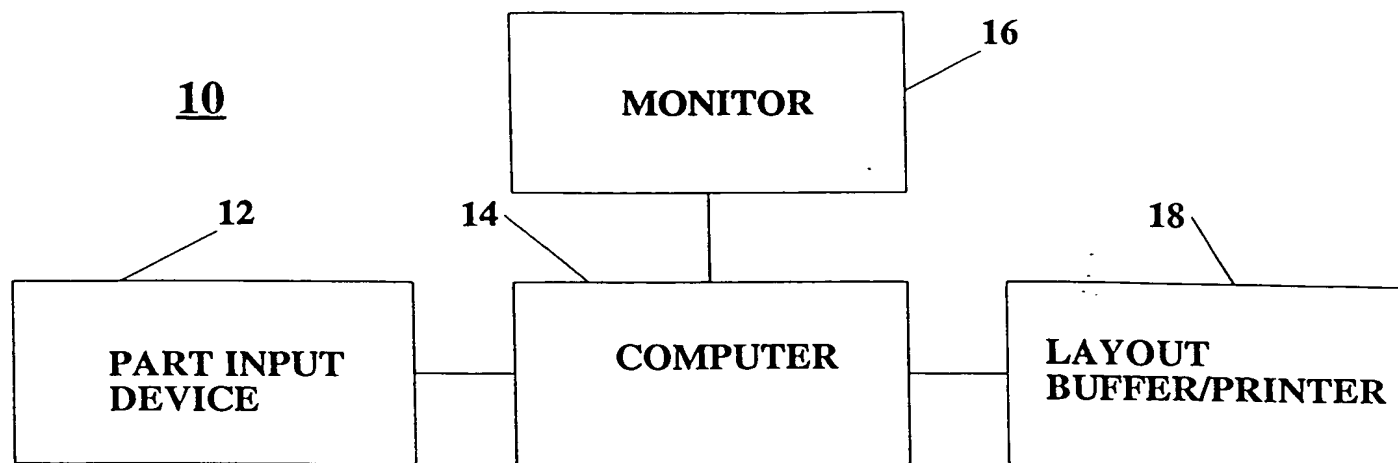


FIG. 1

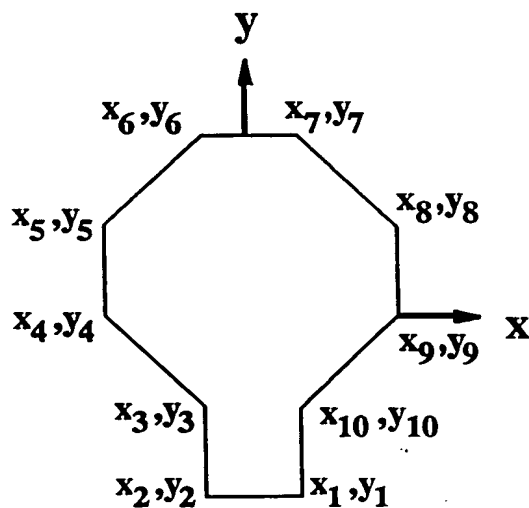


FIG. 2

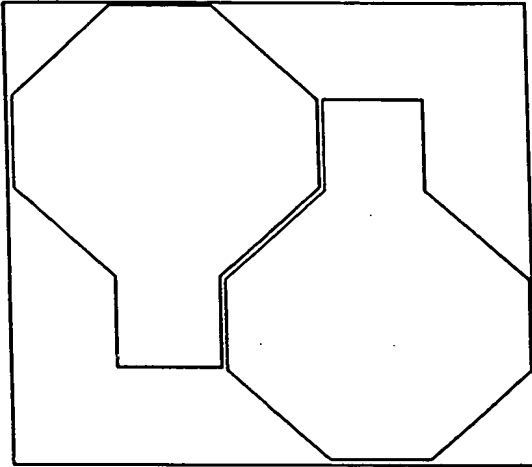


FIG. 3

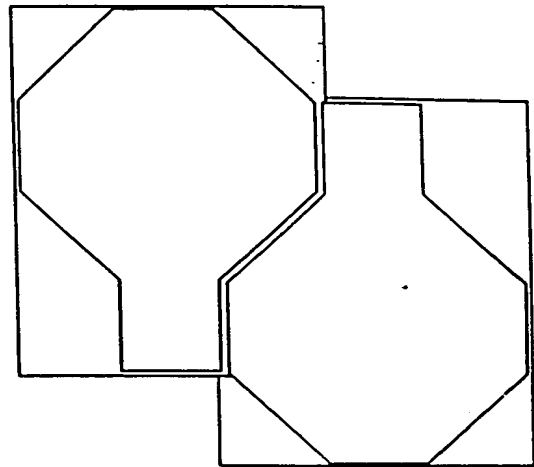


FIG. 4

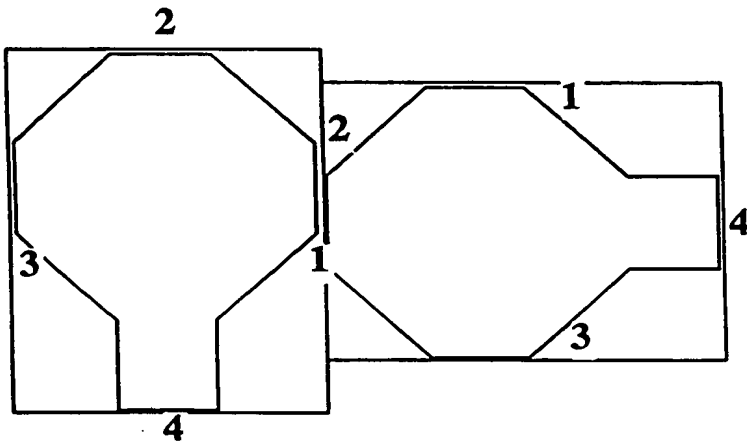


FIG. 5A

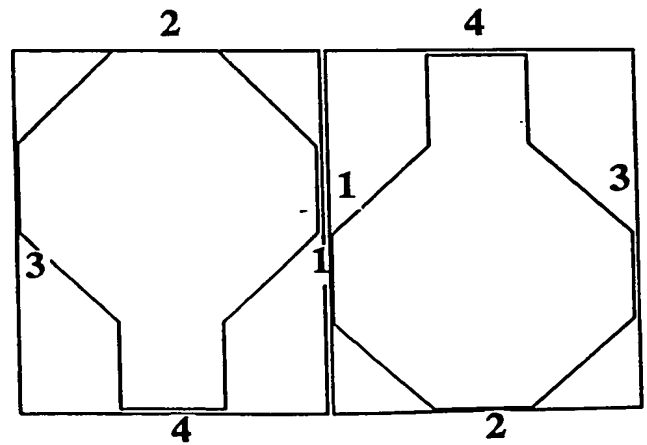


FIG. 5B

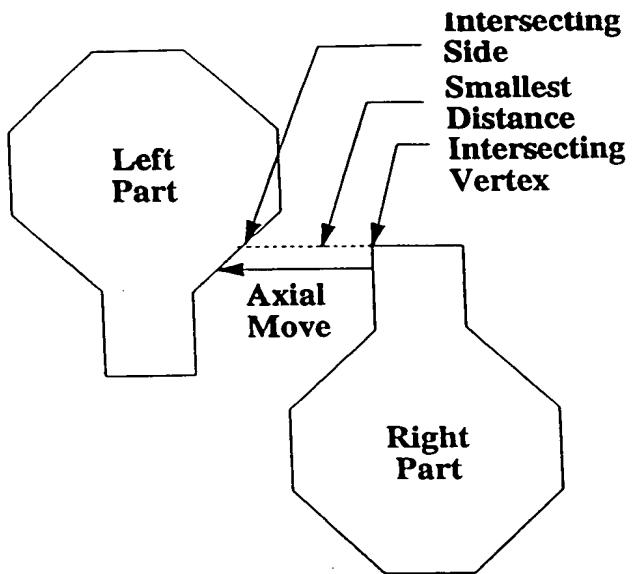


FIG. 6A

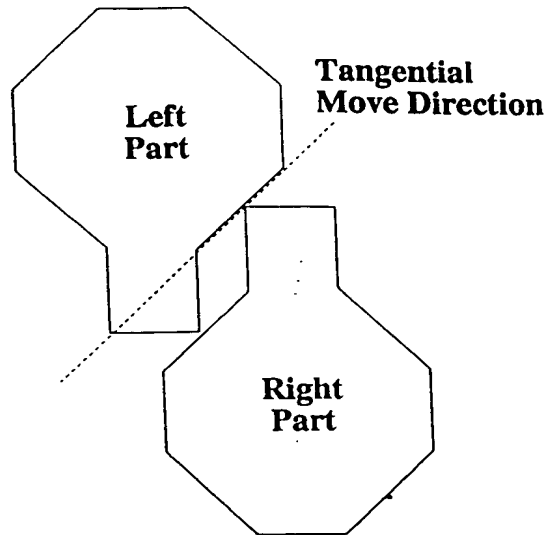


FIG. 6B

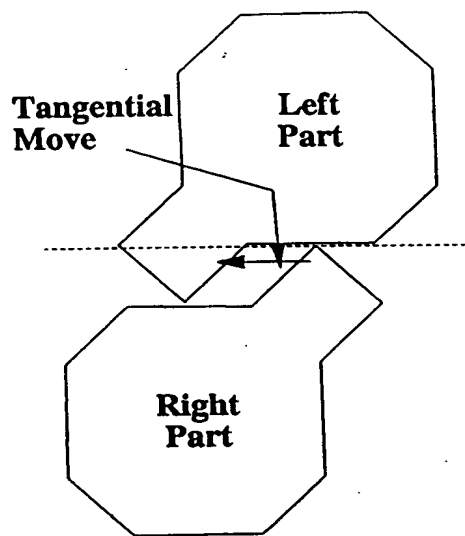


FIG. 6C

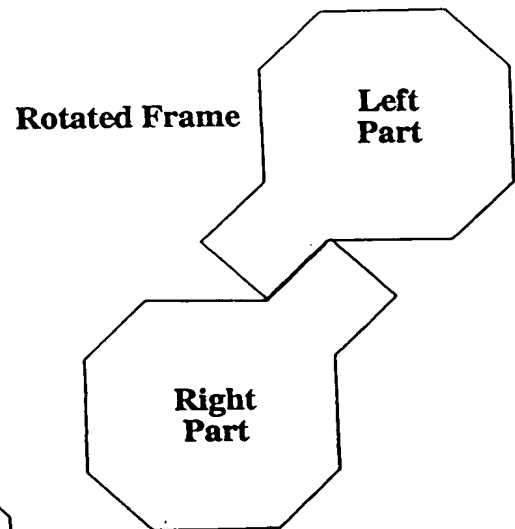


FIG. 6D

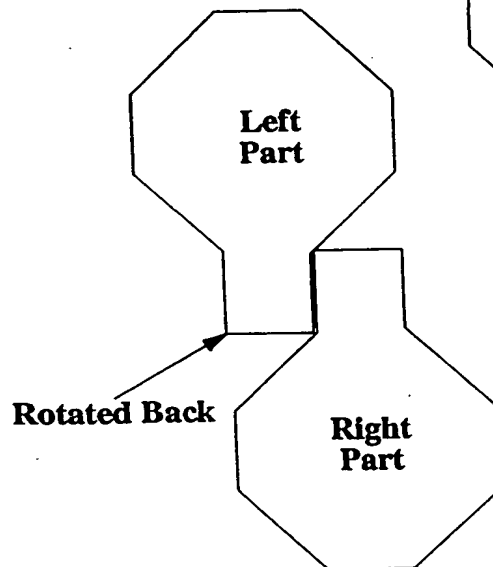
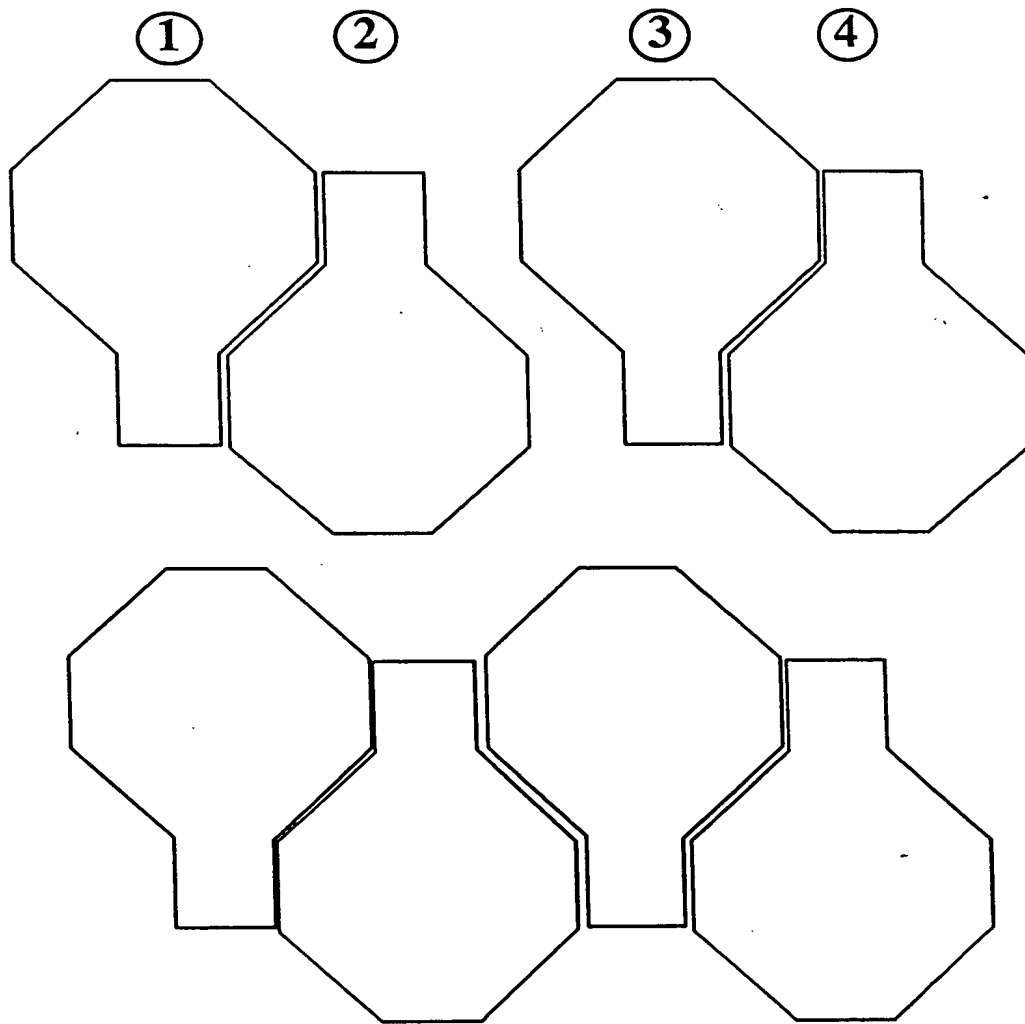


FIG. 6E



**FIG. 7**

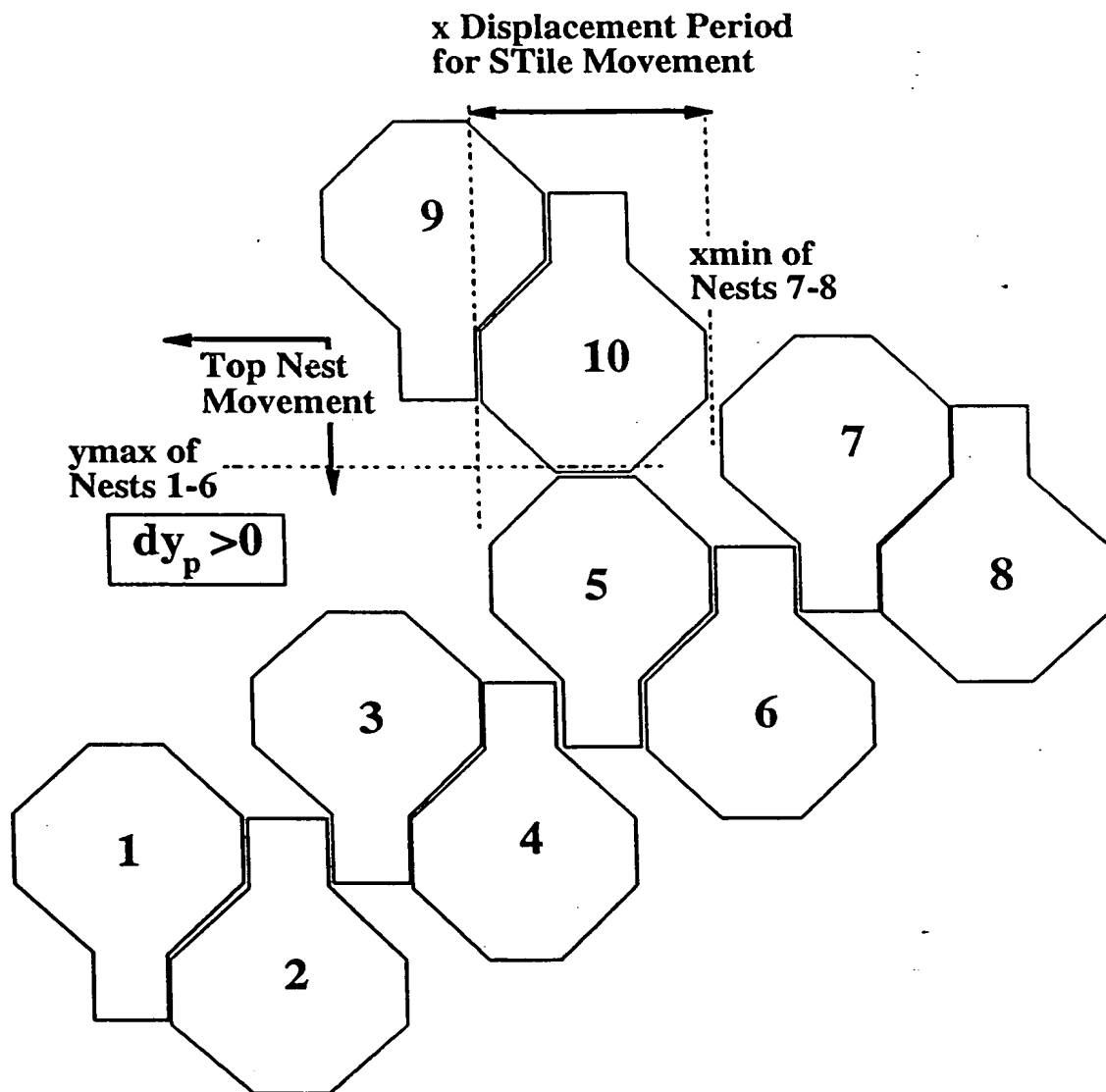


FIG. 8

Ideal Utilization Ratio = .8397  
 REMedEyestay  
 Target Utilization Ratio = .7336  
 for a 3m X 1m Sheet  
 Target Utiliz for a 5m X 1m = .7612

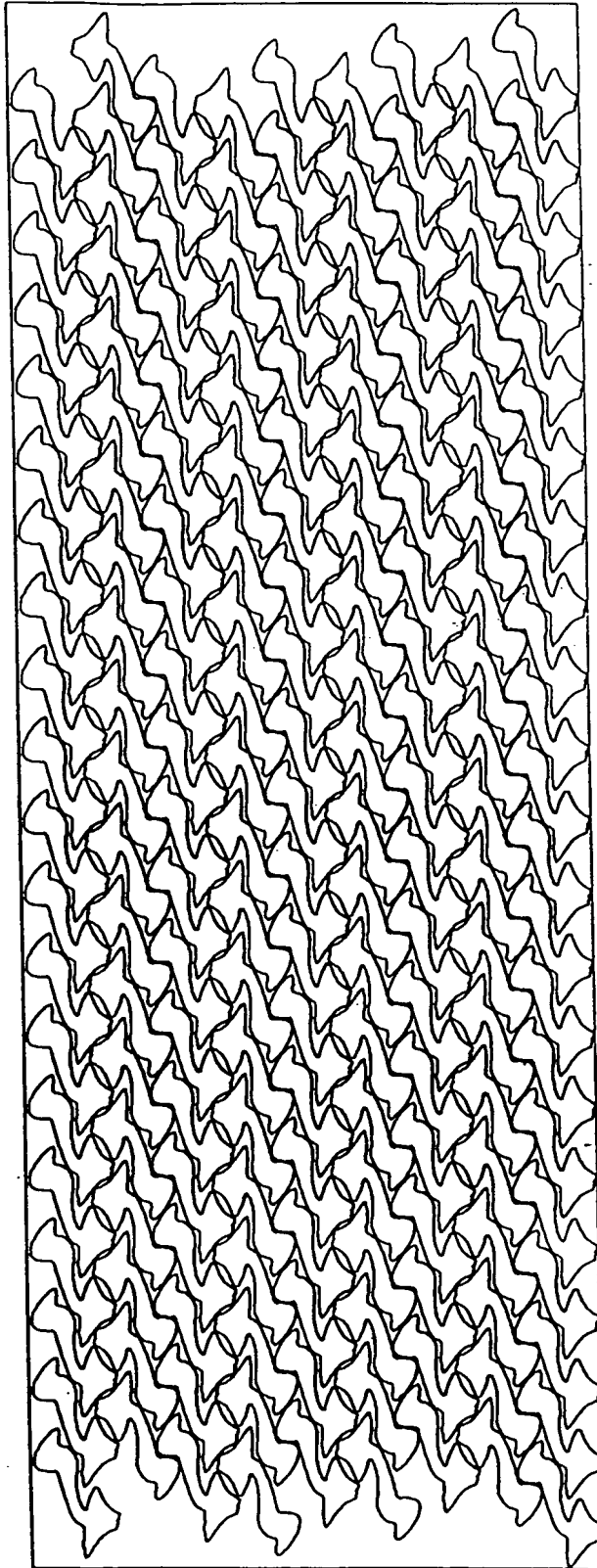


FIG.9



Ideal Utilization Ratio = .8201  
 RBTlp RBTlp  
 Target Utilization Ratio = .7813  
 for a 3m X 1m Sheet  
 Target Utiliz for a 5m X 1m = .7943

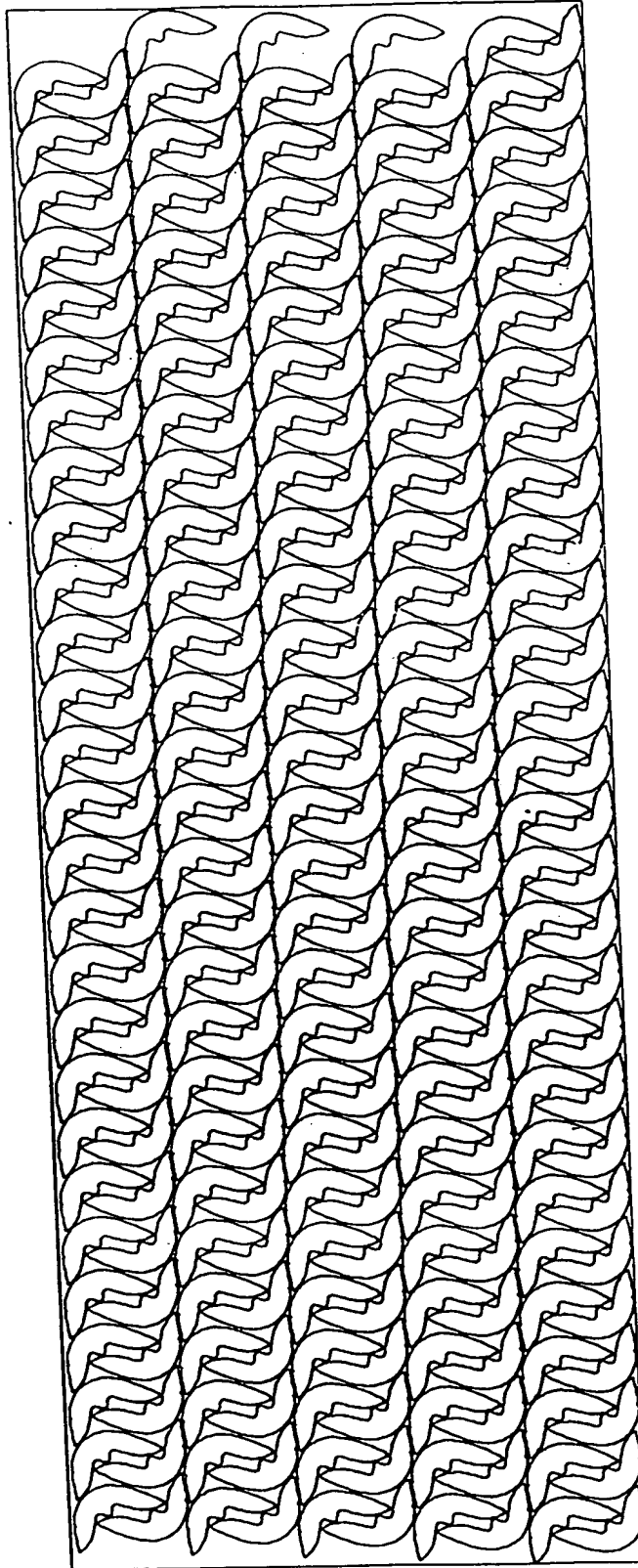


FIG. 10

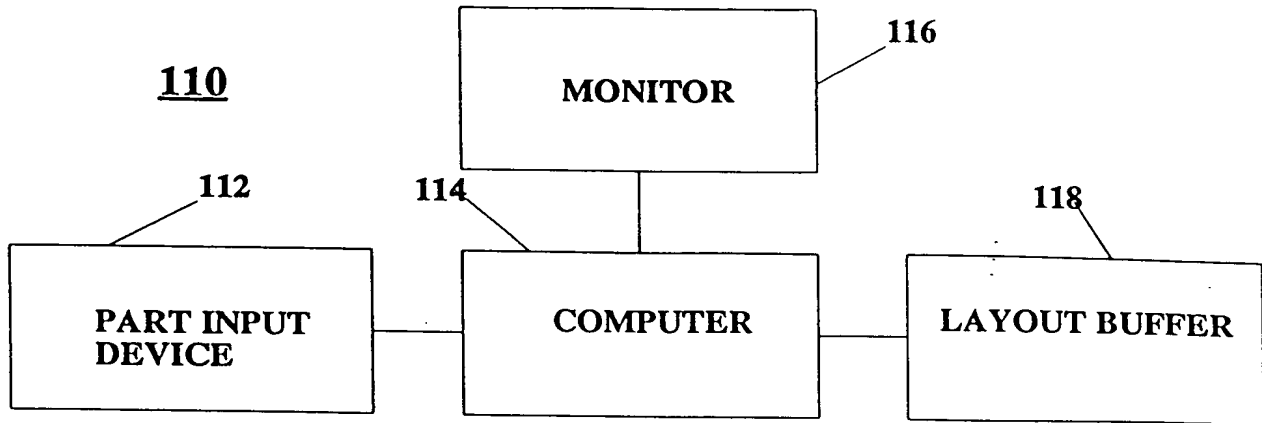


FIG. 11

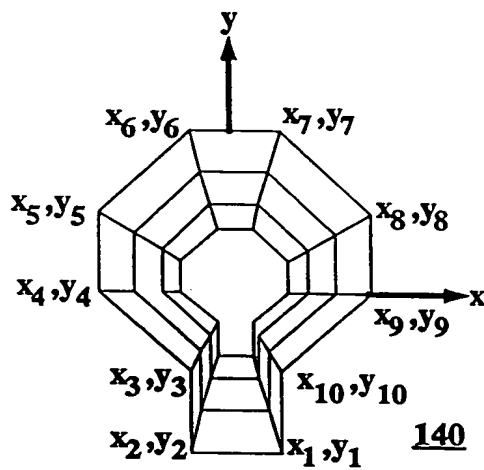


FIG. 12A

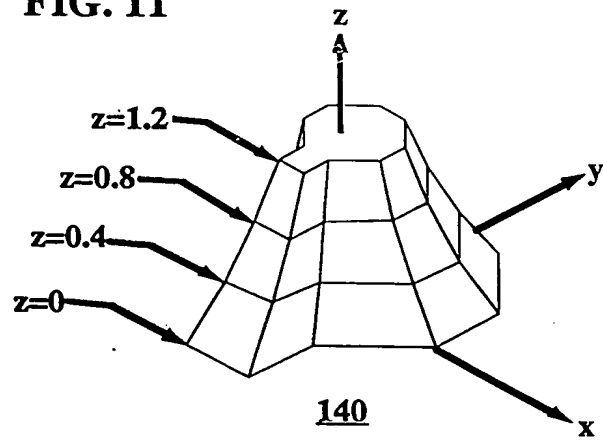


FIG. 12B

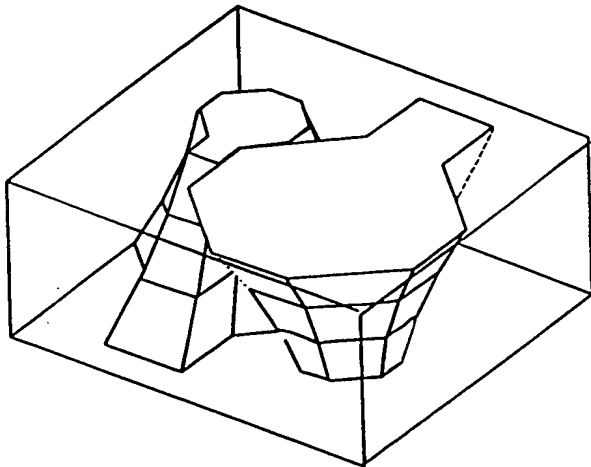


FIG. 13

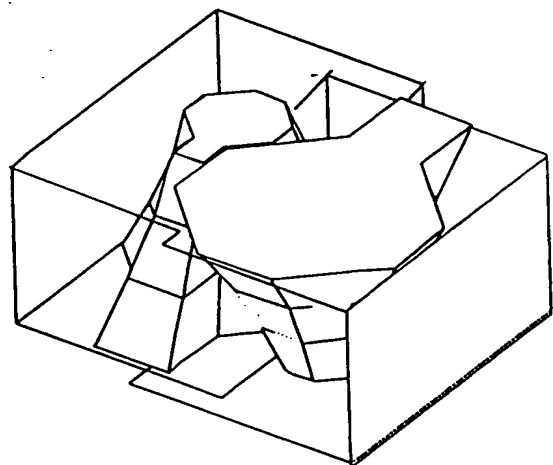


FIG. 14

IPEA/US 12 JUL 1999

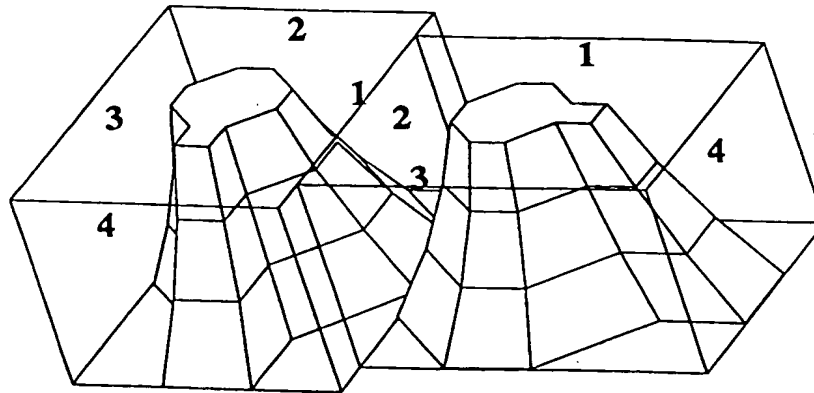


FIG. 15A

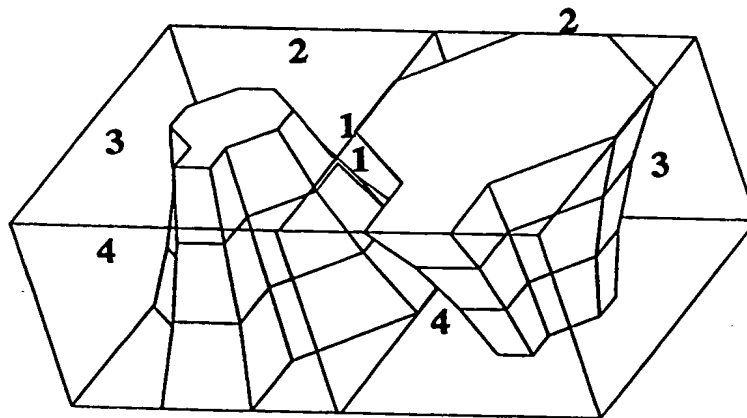


FIG. 15B

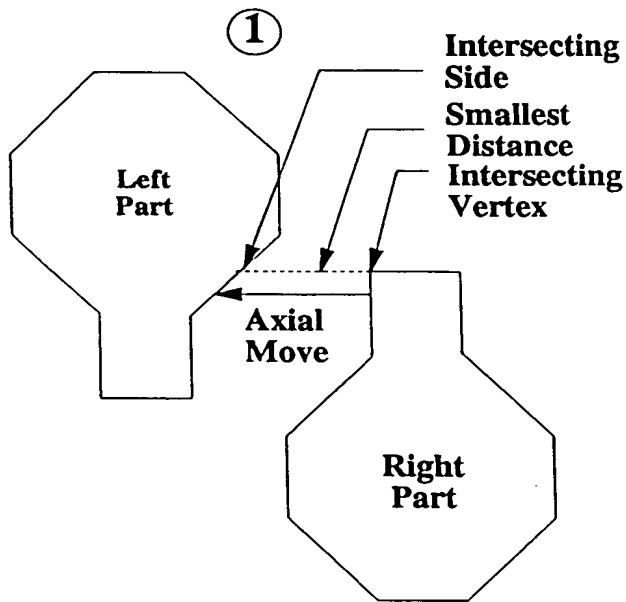


FIG. 16A

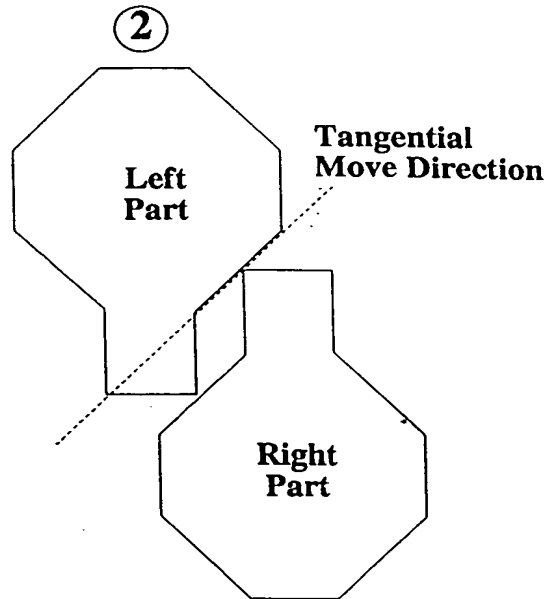


FIG. 16B

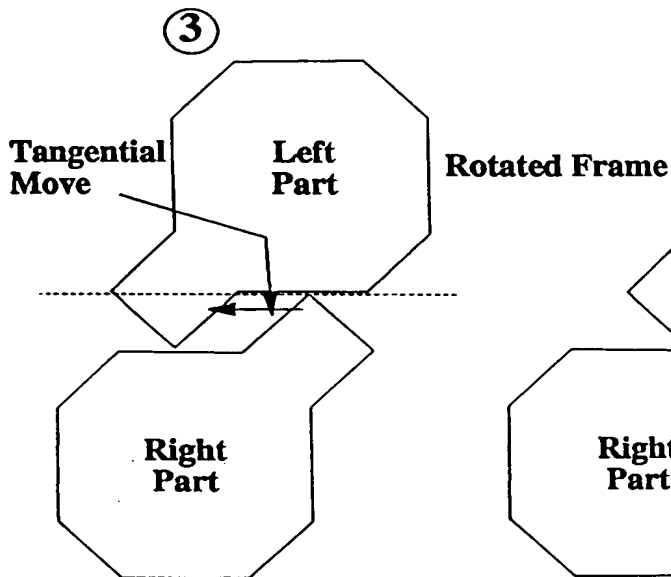


FIG. 16C

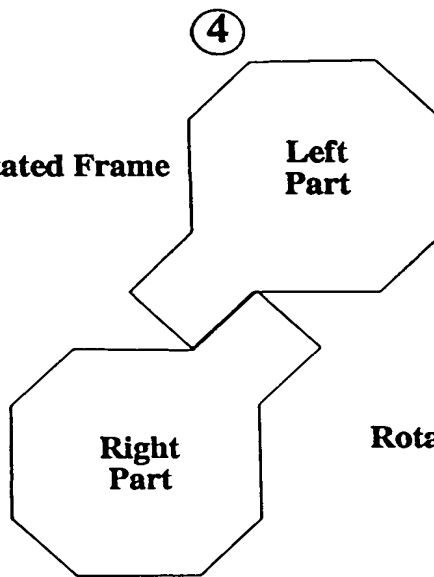


FIG. 16D

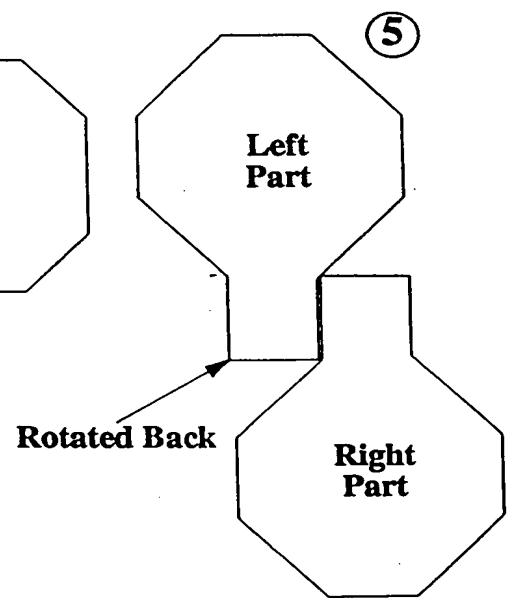
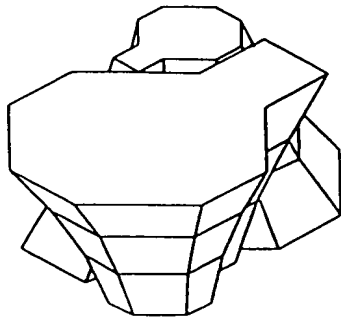
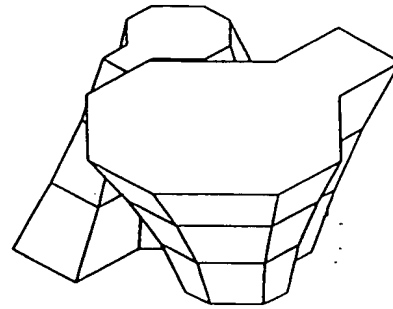


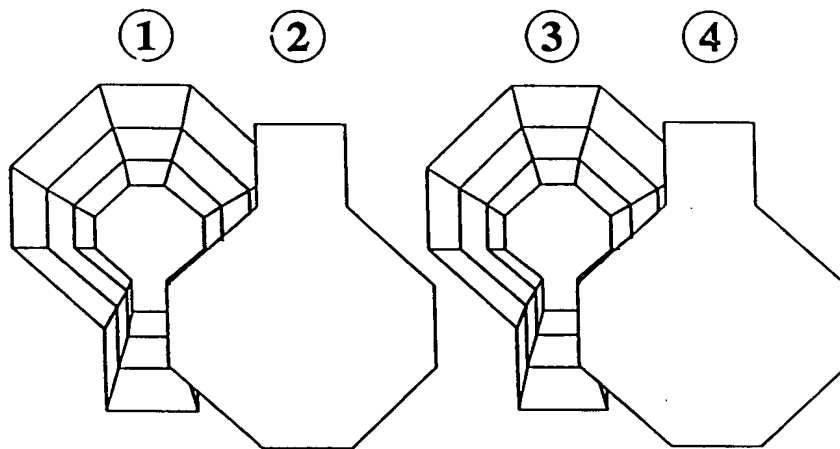
FIG. 16E



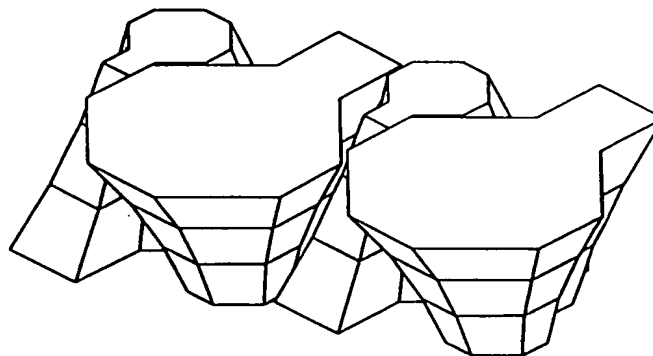
**FIG. 17A**



**FIG. 17B**



**FIG. 18A**



**FIG. 18B**

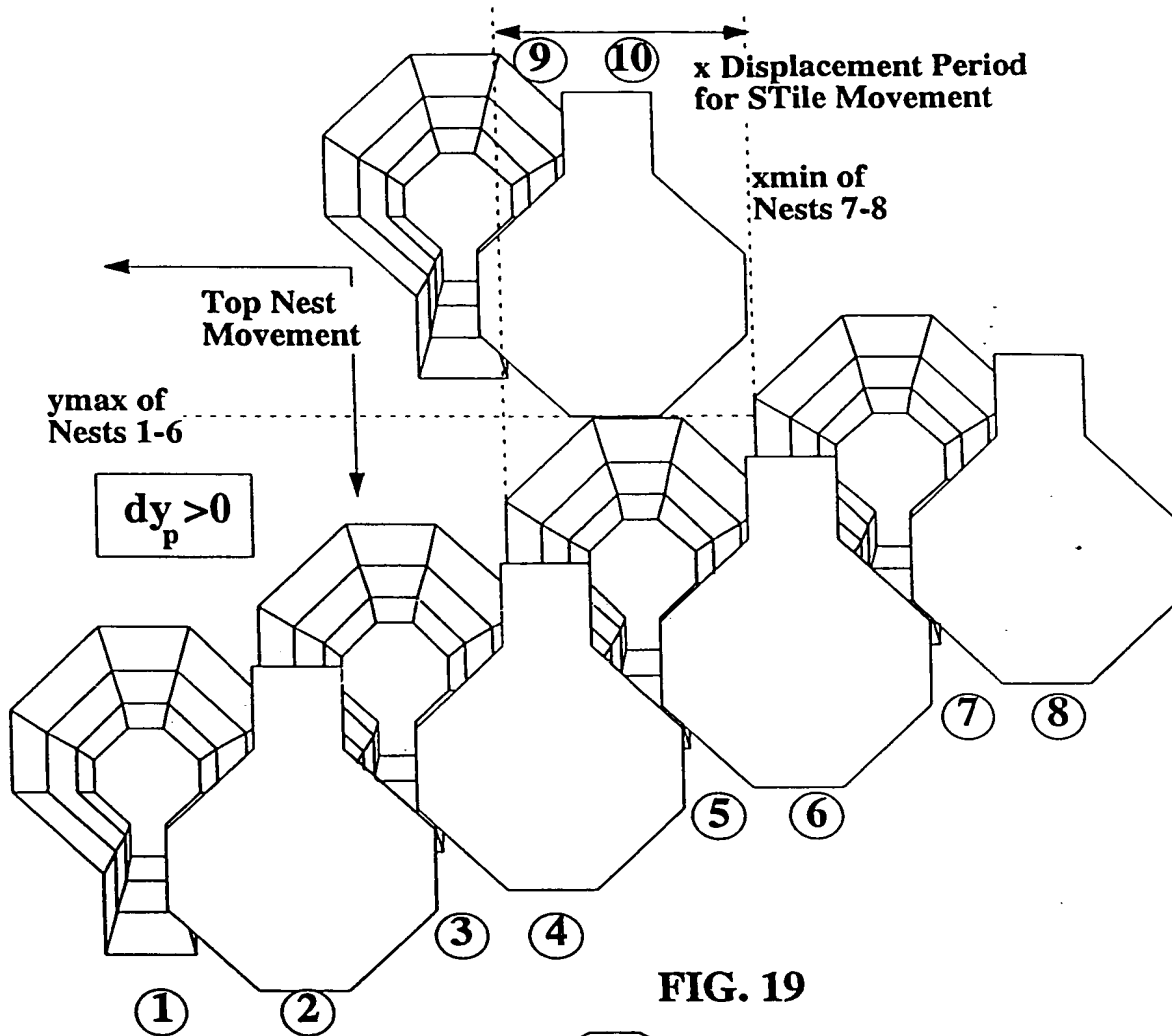


FIG. 19

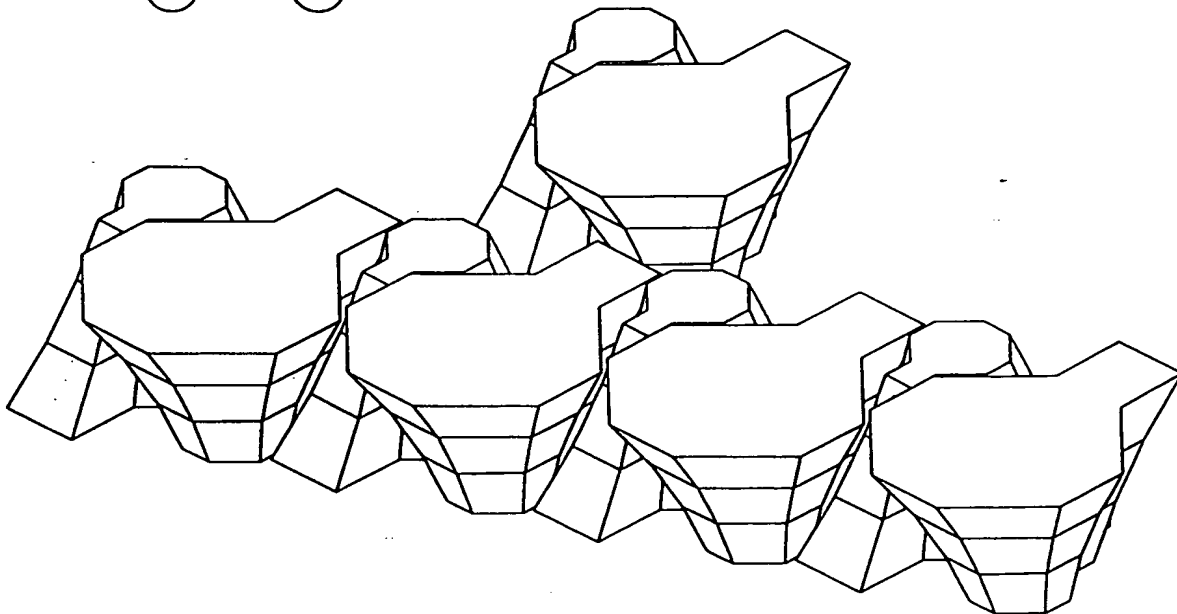


FIG. 20

Number of Nests/Ptiles Kept = 5  
Utilization Ratio = 0.346

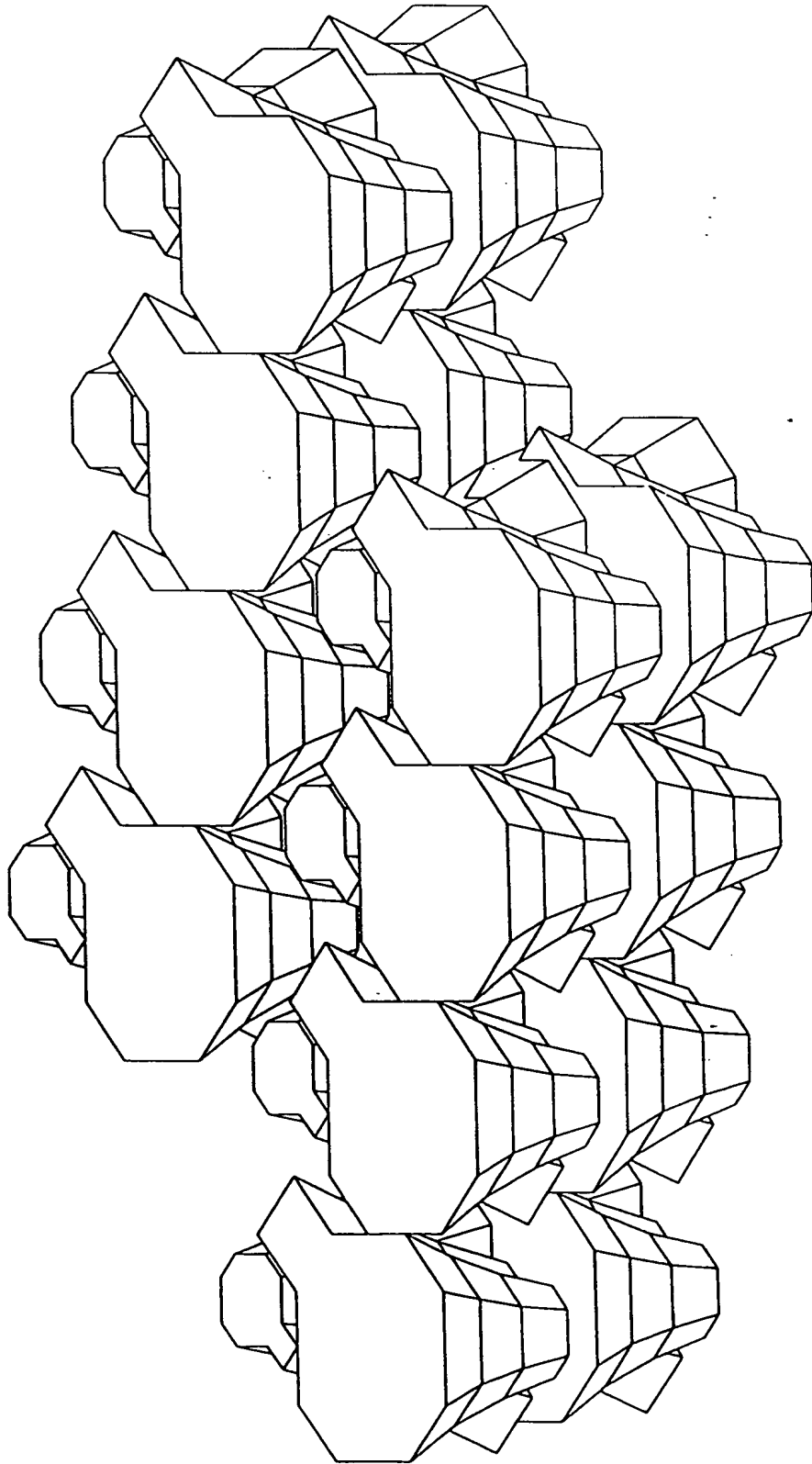


FIG. 21

Number of Nests/Piles Kept = 10  
Utilization Ratio = 0.406

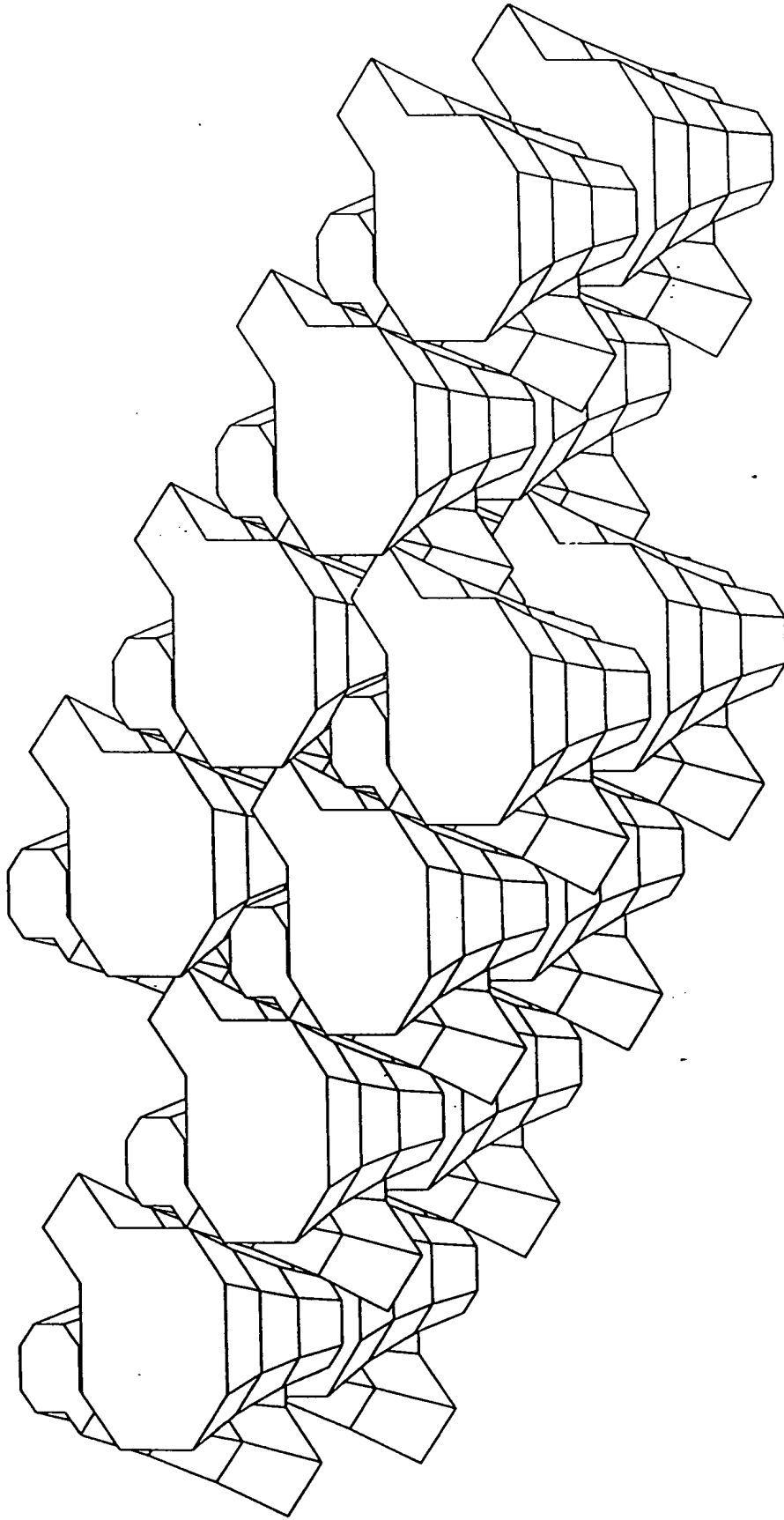


FIG. 22



Number of Nests/PTiles Kept = 20  
Utilization Ratio = 0.422

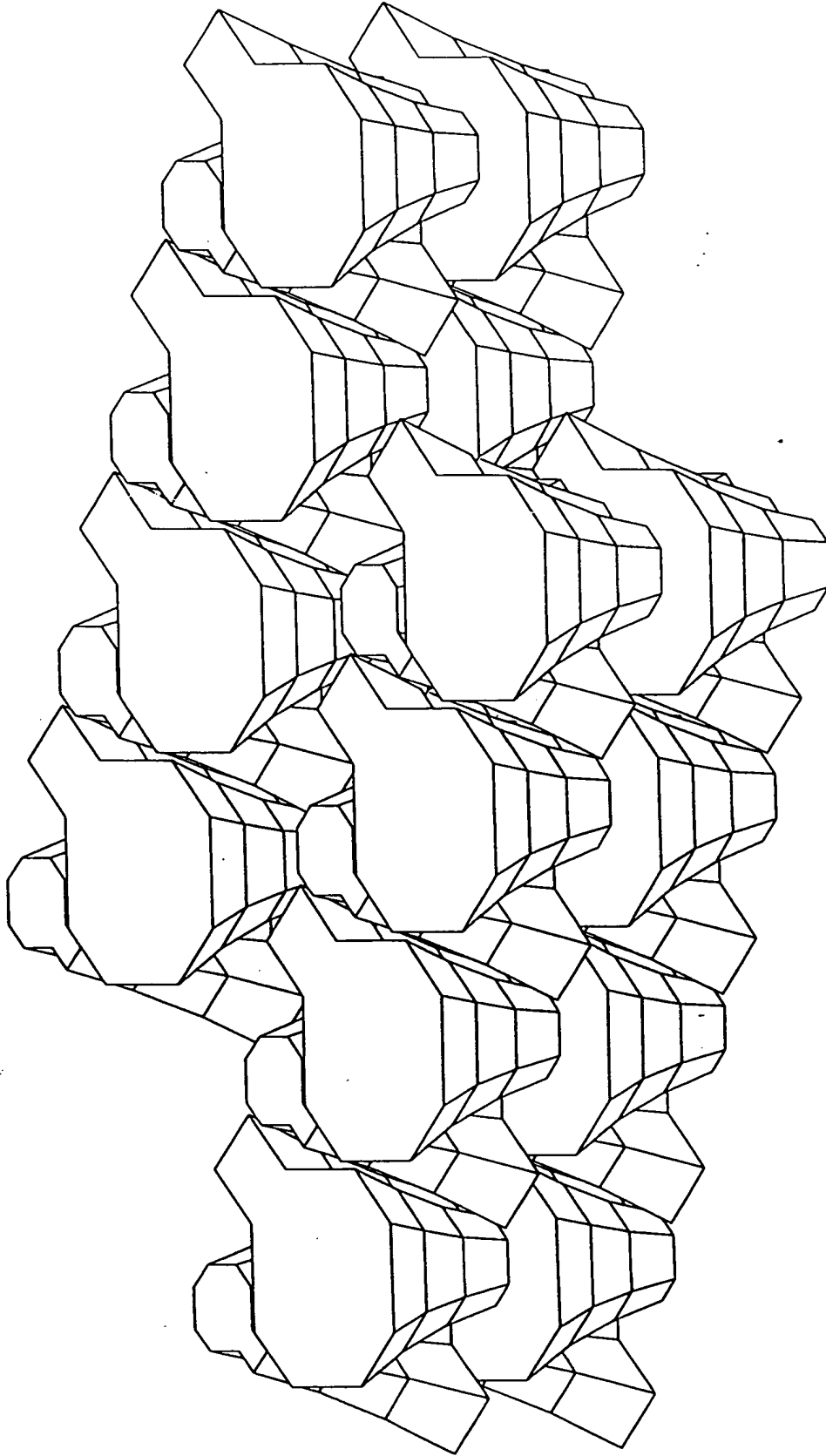


FIG. 23